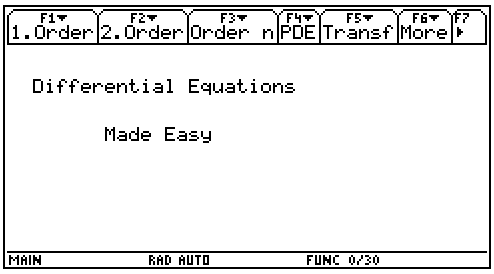
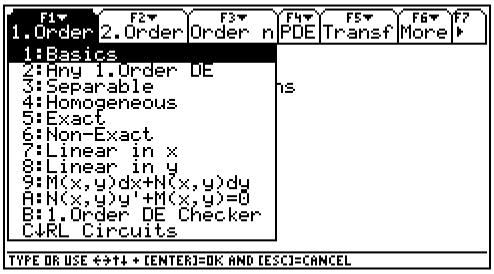
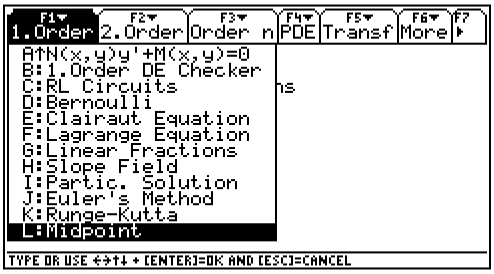
Overview of

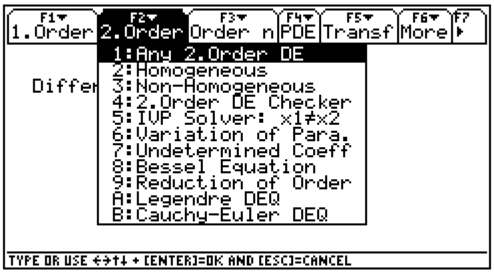
Differential Equations Made Easy – F1 Menu

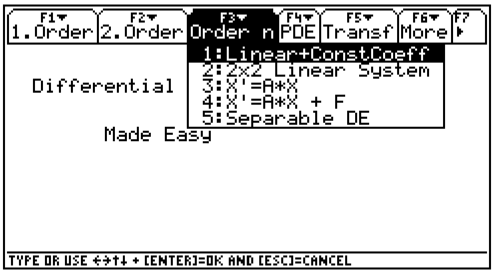
First of all, see some screen shots which show all options offered in the various menus reaching from Basics of 1[st] order DEs over PDEs to special DEs, *Laplace* Transforms and Eigenvalues.

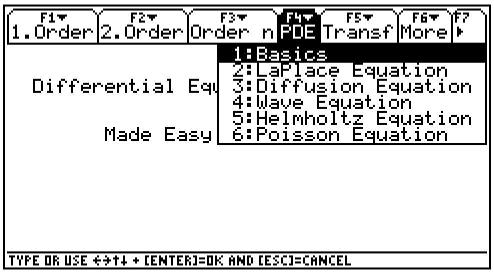


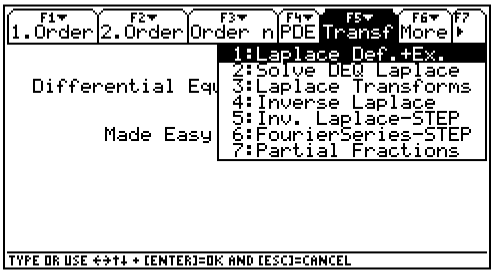


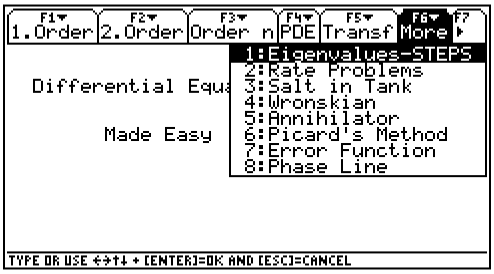




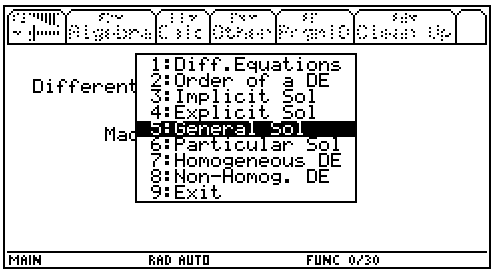


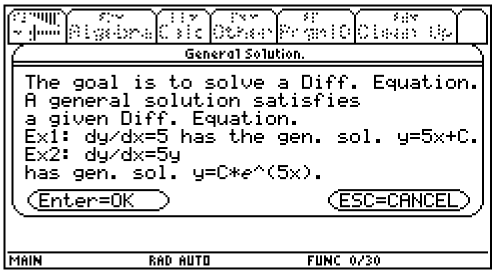




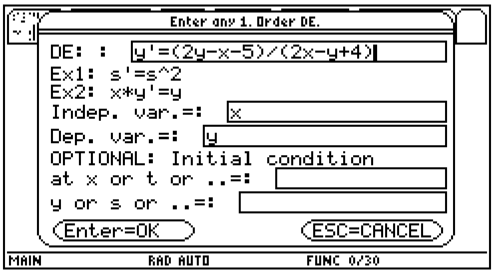


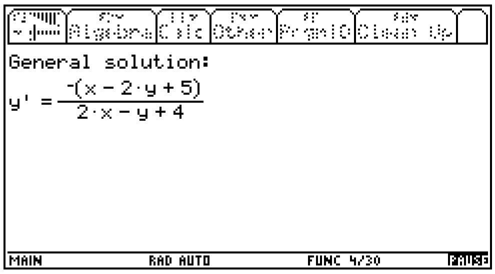
I open the first menu point under F1 1.Order, 1:Basics and want to inform about General Sol(ution):



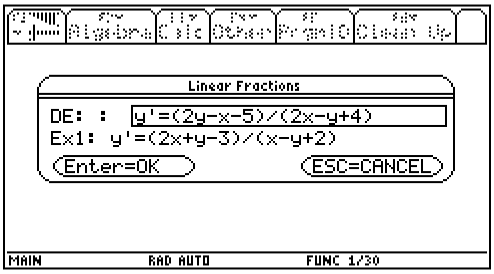


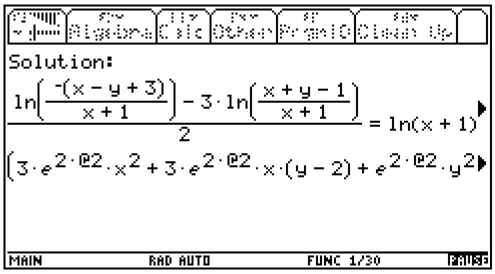
Then I switch back to F1, 2:Any 1.Order DE and would like to solve the differential equation $y' = \dfrac{2y - x - 5}{2x - y + 4};\ y(0) = 4$.



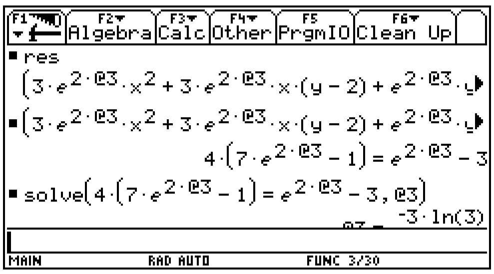


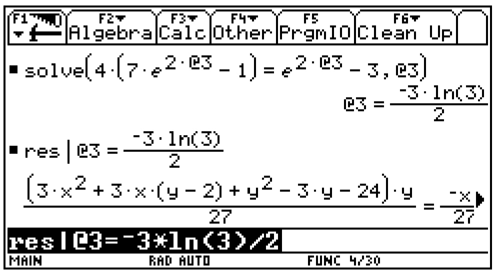
I don´t receive the solution because the TI built in desolve cannot solve this kind of DE. Under F1 I can find the option G:Linear Fractions. So I try again:

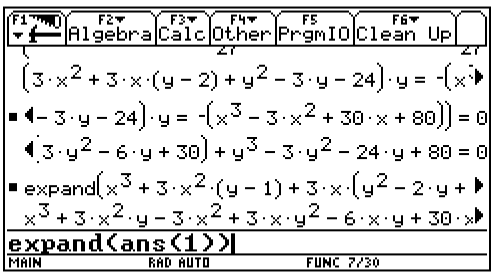




This is the general solution, but how to obtain the special solution? The solution which is presented in the Prgm IO screen is stored as res and can be recalled in the Home screen. In F7 Menu you can find the respective note.







Just for checking the result I load DERIVE and apply the `LIN_FRAC`-function:

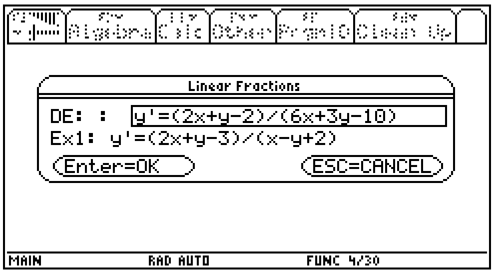
$$\mathrm{LIN\_FRAC}\left(\frac{2\cdot y-x-5}{2\cdot x-y+4},\ -1,\ 2,\ -5,\ 2,\ -1,\ 4,\ x,\ y,\ 0,\ 4\right)$$

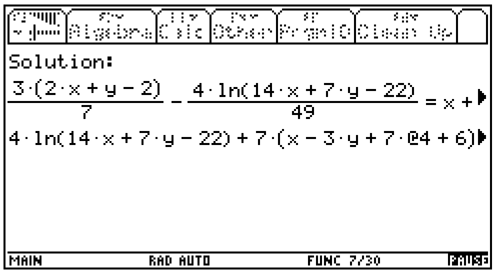
$$\frac{\mathrm{LN}\left(-\frac{x-y+3}{x+1}\right)}{2}-\frac{3\cdot\mathrm{LN}\left(\frac{x+y-1}{x+1}\right)}{2}=\mathrm{LN}(x+1)-\frac{3\cdot\mathrm{LN}(3)}{2}$$

After some manipulations I obtain the same solution! Well done, DEQME!!

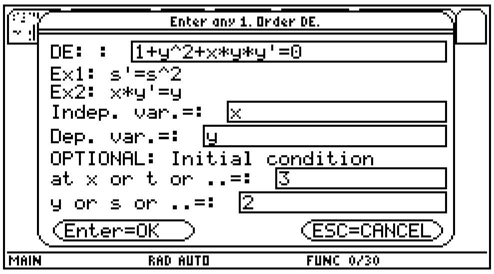
$$\#11:\quad x^3+3\cdot x^2\cdot y-3\cdot x^2+3\cdot x\cdot y^2-6\cdot x\cdot y+30\cdot x+y^3-3\cdot y^2-24\cdot y=-80$$

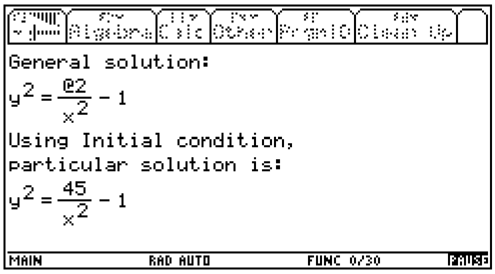
In case of non intersecting linear functions in numerator and denominator DERIVE provides a special utility function `FUN_LIN_CFF_GEN`. This special case was implemented as you can see below:



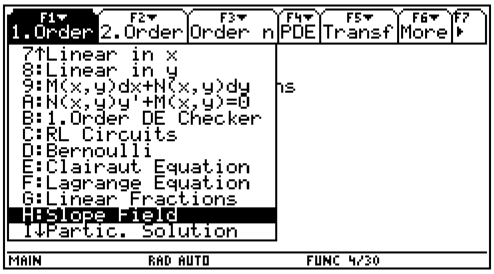


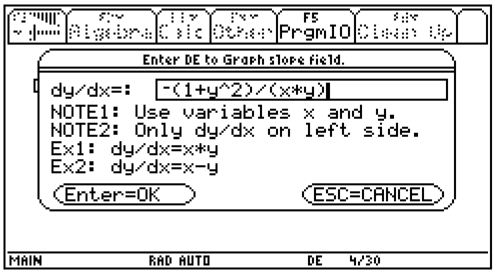
I try solving another DE using the first menu point "Any 1. Order DE"
$1+y^2+x\cdot y\cdot y'=0;\ y(3)=2.$



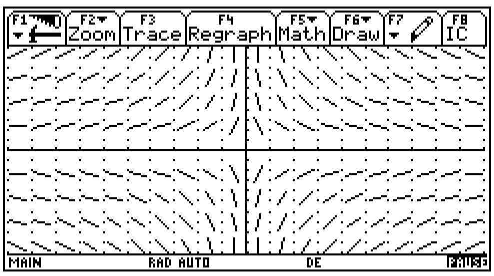


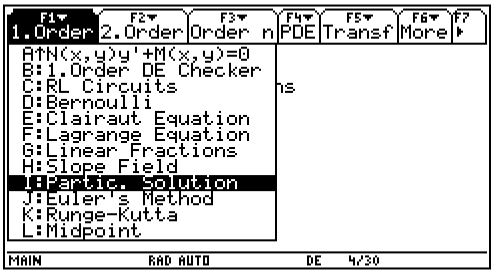
And I'd like to plot its slope field and the special solution.

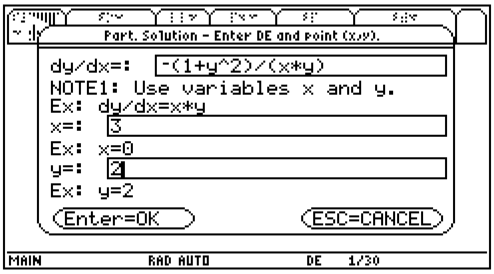


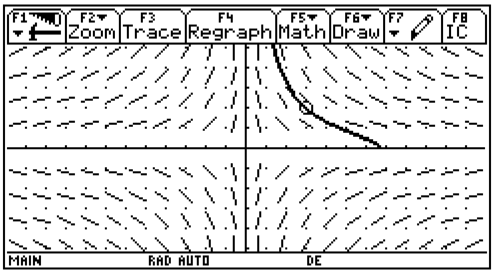


As you can read in the bottom line, the GRAPH Mode changed from FUNC to DE.

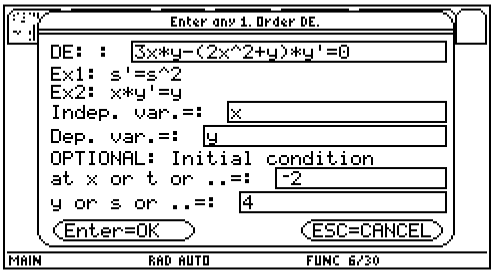


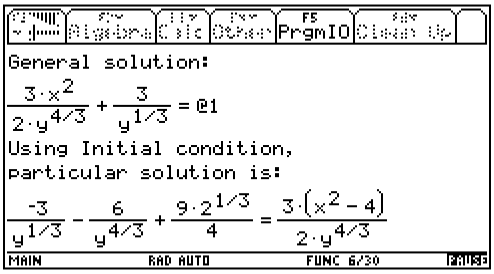


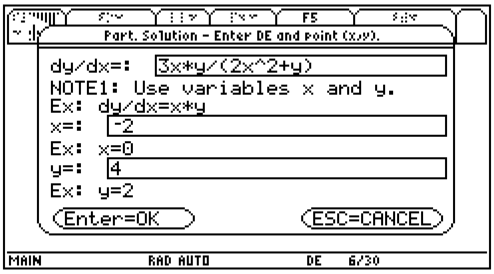


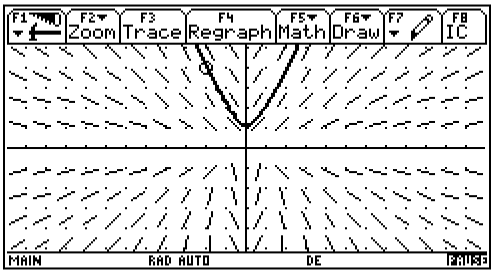


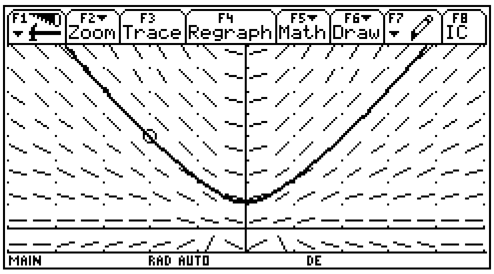
$y' = \dfrac{3xy}{2x^2 + y}$, $y(-2) = 4$. Solve and plot the solution together with the direction field.



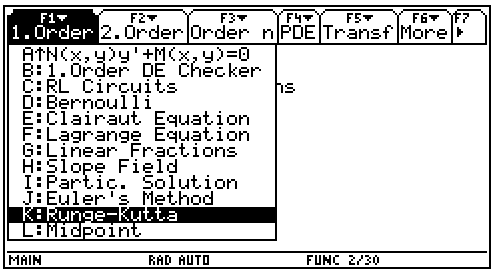


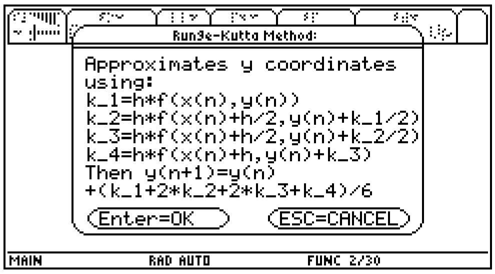


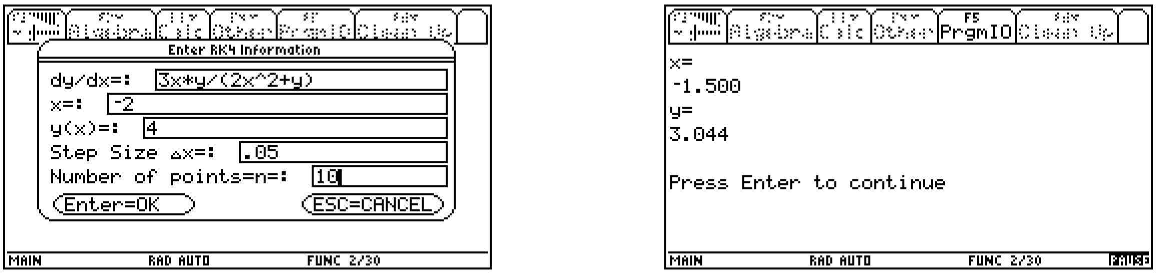




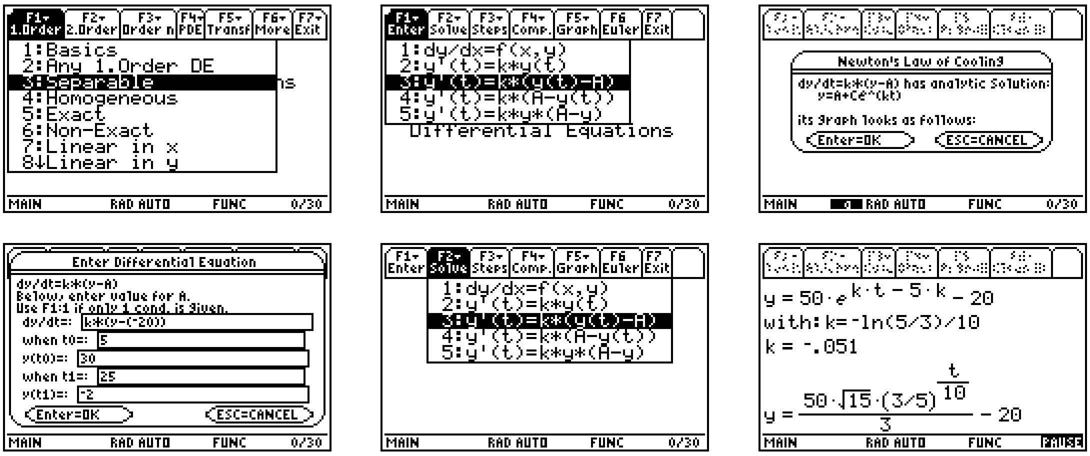
Runge-Kutta is implemented in order to find numerical solutions.


F1 1.Order
F2 2.Order
F3 Order n
F4 PDE
F5 Transf
F6 More
F7
A:N(x,y)y'+M(x,y)=0
B:1.Order DE Checker
C:RL Circuits
D:Bernoulli
E:Clairaut Equation
F:Lagrange Equation
G:Linear Fractions
H:Slope Field
I:Partic. Solution
J:Euler's Method
K:Runge-Kutta
L:Midpoint
MAIN
RAD AUTO
FUNC 2/30

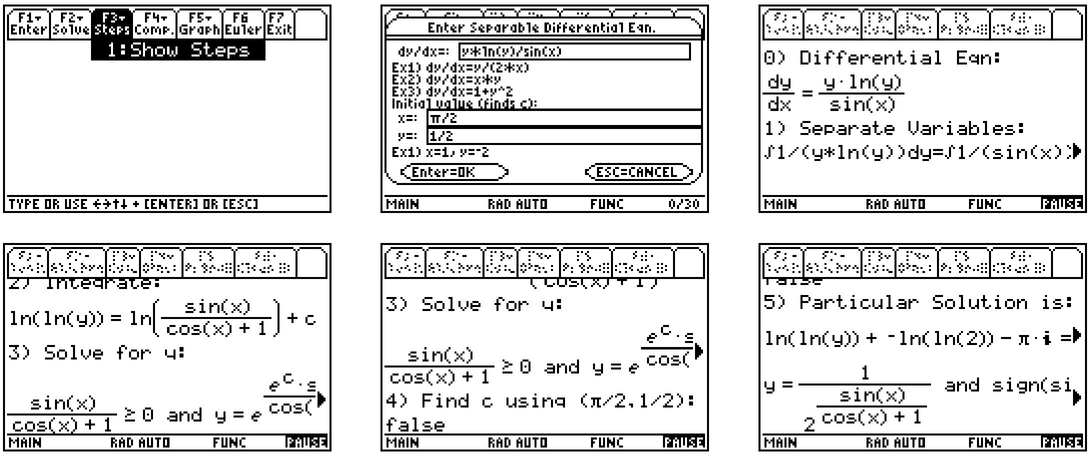

Runge-Kutta Method:
Approximates y coordinates
using:
k_1=h*f(x(n),y(n))
k_2=h*f(x(n)+h/2,y(n)+k_1/2)
k_3=h*f(x(n)+h/2,y(n)+k_2/2)
k_4=h*f(x(n)+h,y(n)+k_3)
Then y(n+1)=y(n)
+(k_1+2*k_2+2*k_3+k_4)/6
Enter=OK
ESC=CANCEL
MAIN
RAD AUTO
FUNC 2/30

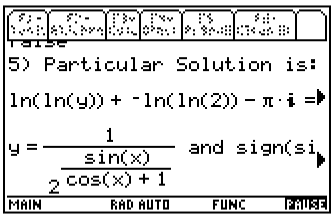


Next examples deal with separation of variables – and I switch to the TI-89:

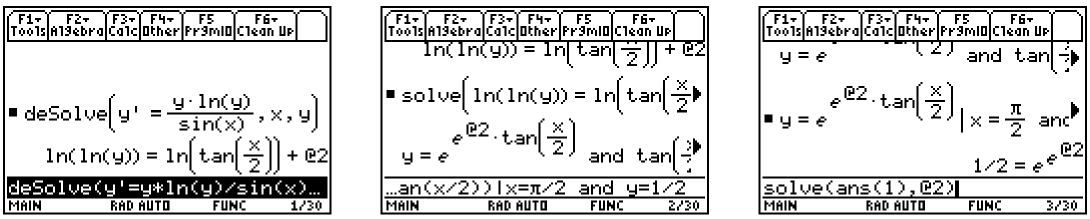


I´d like to see the calculation steps:



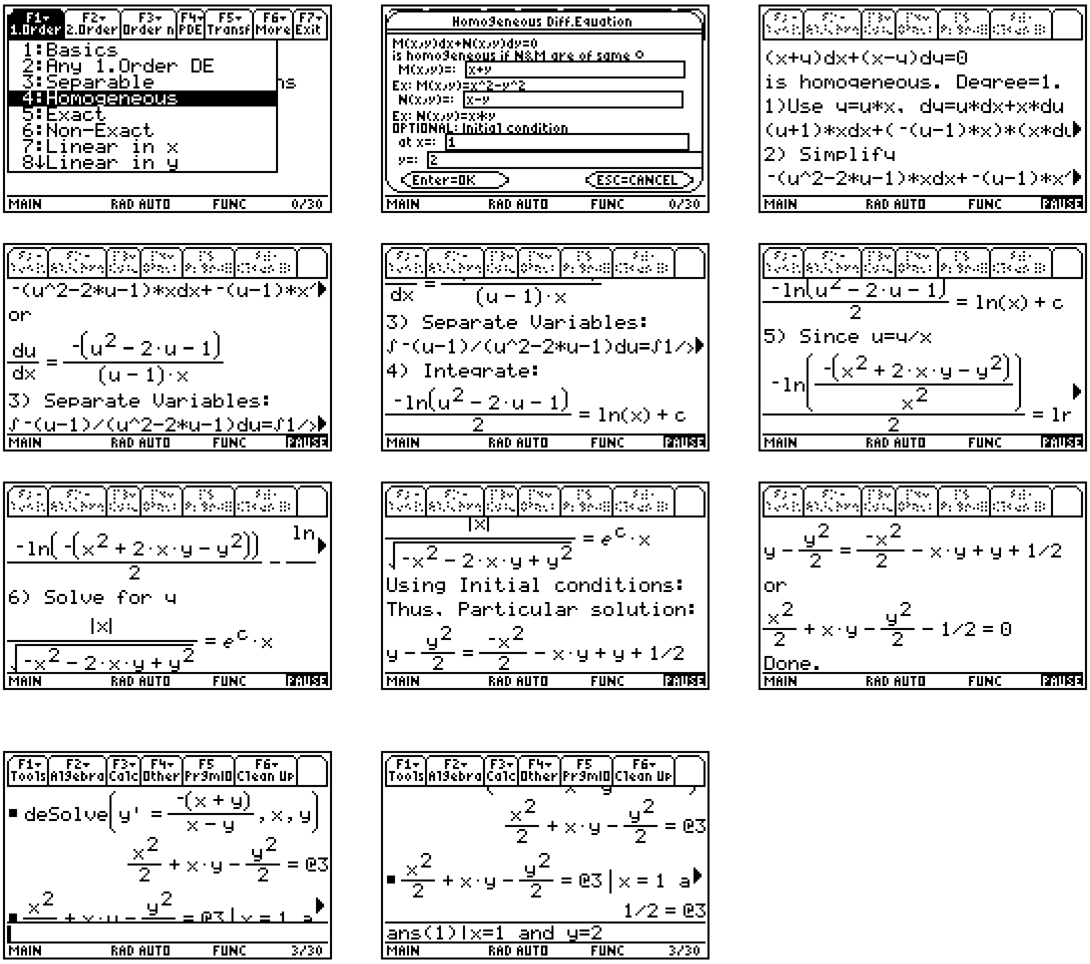


Compare with the deSolve – result. It might be a nice problem for students to verify the identity of the trig expressions.

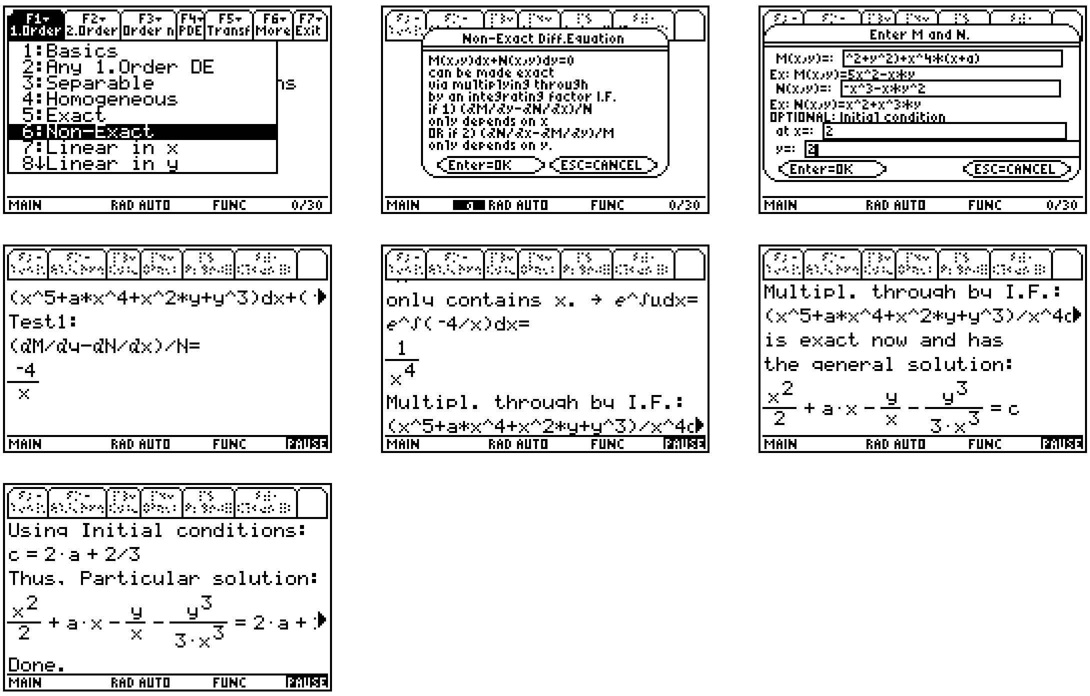


$y' = \dfrac{x+y}{y-x}$; $y(1)=2$. This is a homogeneous DE.

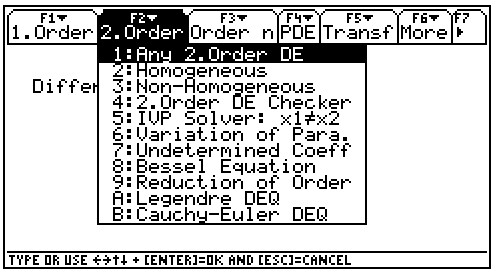
Again I´d like to I follow the steps. Then I will compare with deSolve.



How to find an integrating factor–- if there is any? DEQME gives the answer:



Differential Equations Made Easy – F2 Menu



I'll try to address all Menu options from 1: through B: using the occasion to demonstrate parallel how to apply *DERIVE* for solving the problems and comparing with the solutions given by other CASs. Most of the examples are from a textbook *Differential Equations*[1].

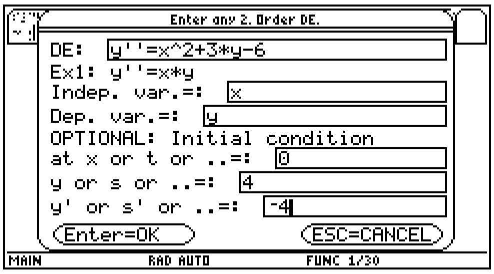
I´ll start with 1: Any 2.Order DE and observe the reaction of *DEQME*. My first package of differential equations is:

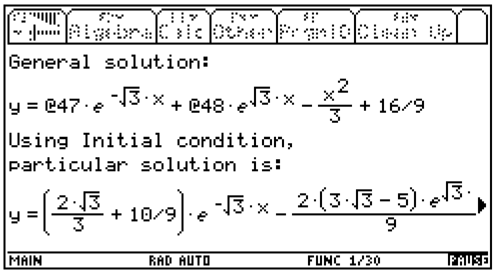
(1) $y'' = x^2 + 3y - 6;\ \ y(0) = 4,\ y'(0) = -4$

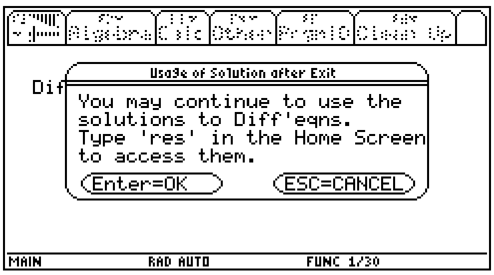
(2) $y'' + 4y = 12x;\ \ y(0) = 5,\ y'(0) = 7$

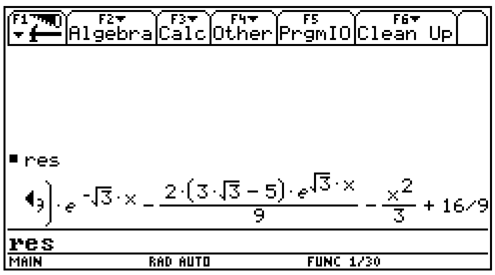
(3) $y'' + y = 2\sin(x)\cdot\sin(2x)$

Example (1)









In order to make comparing the tools easier I will do that example for example. I will am starting with *DERIVE* and proceed with *WIRIS*, *wxMaxima*, *MuPAD*, and with *TI-Nspire*, too, of course.

DERIVE's Online Help informs about the syntax for solving 2nd order ODEs:

DSOLVE2(p, q, r, x, c1, c2) simplifies to an explicit general solution of the linear second order ordinary differential equation

$$y'' + p(x)\cdot y' + q(x)\cdot y = r(x)$$

DSOLVE2_BV(p, q, r, x, x0, y0, x2, y2) is similar to DSOLVE2, but simplifies to a specific solution that satisfies the boundary conditions y=y0 at x=x0 and y=y2 at x=x2.

DSOLVE2_IV(p, q, r, x, x0, y0, v0) is similar toDSOLVE2_BV, but simplifies to a specific solution that satisfies the initial conditions y=y0 and y'=v0 at x=x0.

#1: $\text{DSOLVE2\_IV}(0, -3, 6 - x^2, x, 0, 4, -4)$

#2: $$e^{\sqrt{3}\cdot x}\cdot\left(\frac{26}{9} - \frac{2\cdot\sqrt{3}}{3}\right) + e^{-\sqrt{3}\cdot x}\cdot\left(\frac{2\cdot\sqrt{3}}{3} + \frac{26}{9}\right) + \frac{3\cdot x^2 - 16}{9}$$

Now let *WIRIS* try the job:

prob1 $:=\text{solve}(y''(x)=x^2+3\cdot y(x)-6, y(0)=4, y'(0)=-4);$

prob1 $\rightarrow \left\{\left[y(x)=\left(\frac{2\cdot\sqrt{3}}{3}+\frac{10}{9}\right)\cdot e^{-\sqrt{3}\cdot x}+\left(-\frac{2\cdot\sqrt{3}}{3}+\frac{10}{9}\right)\cdot e^{\sqrt{3}\cdot x}-\frac{x^2}{3}+\frac{16}{9}\right]\right\}$

I applied a “trick” introducing prob1 in order to avoid presenting the equation together with the result in one line which would have been difficult to print it here in a reasonable size.

Next in the row is *wxMaxima*:

(%i1) `eqn_1: 'diff(y,x,2) = x^2+3*y-6;`

(%o1) $$\frac{d^2}{d\,x^2}y = 3\,y + x^2 - 6$$

(%i2) `ode2(%o1, y, x);`

(%o2) $$y = \%k1\,\%e^{\sqrt{3}\,x} + \%k2\,\%e^{-\sqrt{3}\,x} - \frac{3\,x^2-16}{9}$$

(%i17) `ic2(%o2, x=0, y=4, 'diff(y,x)=-4);`

(%o17) $$y = -\frac{(6\sqrt{3}-10)\,\%e^{\sqrt{3}\,x}}{9} + \frac{(6\sqrt{3}+10)\,\%e^{-\sqrt{3}\,x}}{9} - \frac{3\,x^2-16}{9}$$

What about *MuPAD*?

`ivp:=ode({y''(x)=x^2+3*y(x)-6,y(0)=4,y'(0)=-4},y(x))`

$$\text{ode}\left(\left\{y(0)=4,\ y'(0)=-4,\ -3\cdot y(x)-x^2+\frac{\partial^2}{\partial x^2}y(x)+6\right\},\ y(x)\right)$$

`solve(ivp)`

$$\left\{e^{-\sqrt{3}\cdot x}\cdot\left(\frac{2\cdot\sqrt{3}}{3}+\frac{10}{9}\right)-e^{\sqrt{3}\cdot x}\cdot\left(\frac{2\cdot\sqrt{3}}{3}-\frac{10}{9}\right)-\frac{x^2}{3}+\frac{16}{9}\right\}$$

Last but not least see the *TI-Nspire*:

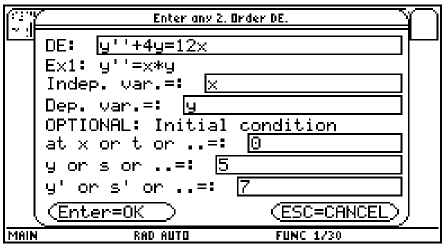
$\text{deSolve}\left(y''=x^2+3\cdot y-6 \text{ and } y(0)=4 \text{ and } y'(0)=-4,x,y\right)$

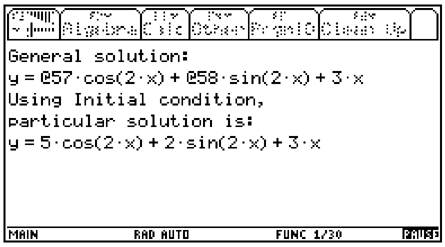
$$y=\left(\frac{2\cdot\sqrt{3}}{3}+\frac{10}{9}\right)\cdot e^{-\sqrt{3}\cdot x}-\frac{2\cdot(3\cdot\sqrt{3}-5)\cdot e^{\sqrt{3}\cdot x}}{9}-\frac{x^2}{3}+\frac{16}{9}$$

TI-Nspire works like the TI-92PLUS and TI-Voyage 200. Please wait, *DEQME* is able to do a lot more than only solving this kind of DEs assisted by a nice form for entering the data.

All systems behave the same so far. I can assure that things will change!

Example (2)





DERIVE:

```
DSOLVE2_IV(0, 4, 12·x, x, 0, 5, 7) = 4·SIN(x)·COS(x) - 10·SIN(x)^2 + 3·x + 5
```

WIRIS:

prob2 := solve(y"(x)+4y(x)=12x,y(0)=5,y'(0)=7);

prob2 → $\{\{y(x) = 6\cdot x\cdot \sin(2\cdot x)^2 + \frac{\sin(2\cdot x)}{2} + 6\cdot x\cdot \cos(2\cdot x)^2 + 5\cdot \cos(2\cdot x)\}\}$

prob2 := simplify$(6\cdot x\cdot \sin(2\cdot x)^2 + \frac{\sin(2\cdot x)}{2} + 6\cdot x\cdot \cos(2\cdot x)^2 + 5\cdot \cos(2\cdot x))$;

prob2 → $5\cdot \cos(2\cdot x) + \frac{\sin(2\cdot x)}{2} + 6\cdot x$

WxMaxima:

(%i20) eqn_2: 'diff(y,x,2)+4*y=12*x;

(%o20) $\frac{d^2}{d\,x^2}y + 4\,y = 12\,x$

(%i21) ode2(%o20,y,x);

(%o21) $y = \%k1\,\sin(2\,x) + \%k2\,\cos(2\,x) + 3\,x$

(%i24) ic2(%o21,x=0,y=5,'diff(y,x)=7);

(%o24) $y = 2\,\sin(2\,x) + 5\,\cos(2\,x) + 3\,x$

MuPAD provides the same result and so do the TIs!

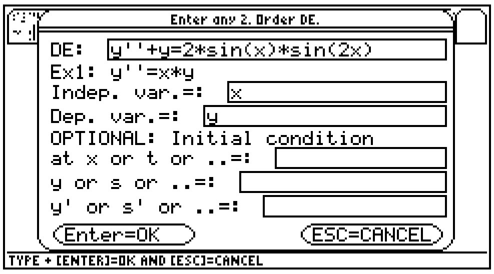
WIRIS needs an extra simplification – it does not simplify $6x\sin^2(2x) + 6x\cos^2(2x)$ to $6x$. The *WIRIS* result seems to be wrong! I am using *WIRIS* itself to check the result – and the right side gives $24x$ instead of $12x$.

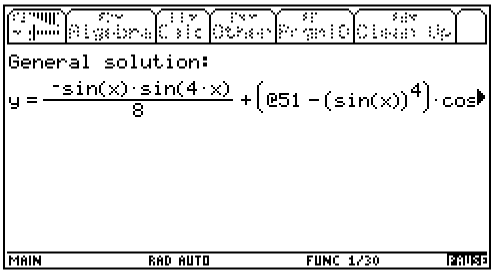
$y(x) := 5\cdot \cos(2\cdot x) + \frac{\sin(2\cdot x)}{2} + 6\cdot x \rightarrow x \mapsto 5\cdot \cos(2\cdot x) + \frac{\sin(2\cdot x)}{2} + 6\cdot x$

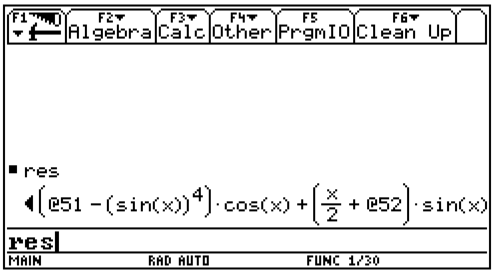
$y''(x) + 4\cdot y(x) \rightarrow 24\cdot x$

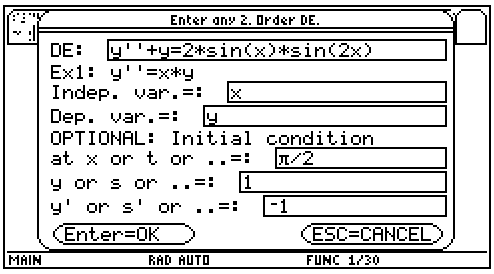
I could have used also *DEQME*’s 2nd order DE Checker to check the presented (correct) solutions!

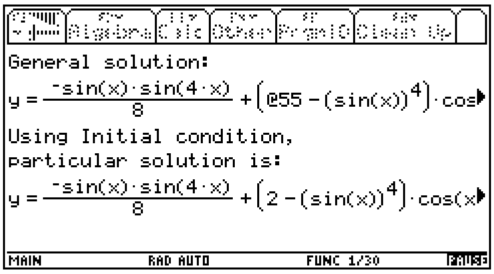
Example (3) (including an additional IVP)**:**

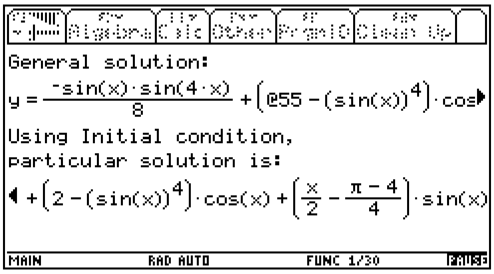












Working with *DERIVE* is interesting:

$$\mathrm{DSOLVE2\_IV}\left(0,\ 1,\ 2\cdot\mathrm{SIN}(x)\cdot\mathrm{SIN}(2\cdot x),\ x,\ \frac{\pi}{2},\ 1,\ -1\right)$$

$$-\frac{\mathrm{COS}(x)\cdot\mathrm{COS}(4\cdot x)}{8}-\frac{\mathrm{SIN}(x)\cdot\mathrm{SIN}(4\cdot x)}{8}+\frac{\mathrm{COS}(x)\cdot\mathrm{COS}(2\cdot x)}{2}+\frac{13\cdot\mathrm{COS}(x)}{8}+\left(\frac{x}{2}-\frac{\pi}{4}+1\right)\cdot\mathrm{SIN}(x)$$

$$\frac{\sqrt{(4\cdot x^2+1)}\cdot\mathrm{COS}(\mathrm{ATAN}(2\cdot x)-x)}{4}-\frac{\sqrt{(4\cdot\pi^2-32\cdot\pi+233)}\cdot\mathrm{SIN}\left(\mathrm{ATAN}\left(\frac{13}{2\cdot(\pi-4)}\right)-x\right)}{8}+\frac{\mathrm{COS}(3\cdot x)}{8}$$

[Trigonometry := Expand, Trigpower := Sines]

$$\mathrm{COS}(x)\cdot\left(2-\frac{\mathrm{SIN}(x)^2}{2}\right)+\left(\frac{x}{2}-\frac{\pi}{4}+1\right)\cdot\mathrm{SIN}(x)$$

There are three different appearances of the result depending on the Trig Mode Settings. You need some skills (or intuiton and luck – just try) to find the appropriate settings in order to obtain a compact form of the solution. Plotting all solutions on the same axes and checking by substituting the solution into the given equation shows the identity of the expressions.

$$y(x) := -\frac{\mathrm{SIN}(x)\cdot\mathrm{SIN}(4\cdot x)}{8}+(2-\mathrm{SIN}(x)^4)\cdot\mathrm{COS}(x)+\left(\frac{x}{2}-\frac{\pi-4}{4}\right)\cdot\mathrm{SIN}(x)$$

$$y''(x)+y(x)-2\cdot\mathrm{SIN}(x)\cdot\mathrm{SIN}(2\cdot x)=0$$

WIRIS presents its solution in a very “extended” form. Plotting and applying the appropriate Trig Mode in *DERIVE* confirms the identity with the other results.

$$\text{solve}\left(y''(x)+y(x)=2\sin(x)\cdot\sin(2x),\ y(\pi/2)=1,\ y'(\pi/2)=-1\right)$$

$$\rightarrow \left\{\left[y(x) = -\frac{\sin(x)\cdot\sin(-2\cdot x)}{4} - \frac{\sin(x)\cdot\sin(4\cdot x)}{8} - \frac{\sin(x)\cdot\sin(2\cdot x)}{4} + \left(\frac{x}{2} - \frac{\pi}{4} + 1\right)\cdot\sin(x)\right.\right.$$

$$y(x) := -\frac{\sin(x)\cdot\sin(-2\cdot x)}{4} - \frac{\sin(x)\cdot\sin(4\cdot x)}{8} - \frac{\sin(x)\cdot\sin(2\cdot x)}{4} + \left(\frac{x}{2} - \frac{\pi}{4} + 1\right)\cdot\sin(x) -$$
$$\frac{\cos(4\cdot x)\cdot\cos(x)}{8} + \frac{\cos(x)\cdot\cos(2\cdot x)}{2} + \frac{13\cdot\cos(x)}{8};$$

This was the first step. Simplifying this result once more delivers the result as above (*DERIVE*):

$$\text{simplify}\left(-\frac{\sin(x)\cdot\sin(-2\cdot x)}{4} - \frac{\sin(x)\cdot\sin(4\cdot x)}{8} - \sin(\ldots\right.$$

$$\rightarrow \left(-\frac{\sin(x)^2}{2} + 2\right)\cdot\cos(x) + \left(\frac{x}{2} - \frac{\pi}{4} + 1\right)\cdot\sin(x)$$

WxMaxima (correct result):

$$y = \frac{\cos(3\,x) + 4\,x\sin(x) - \cos(x)}{8} - \frac{(\%pi - 4)\sin(x)}{4} + 2\cos(x)$$

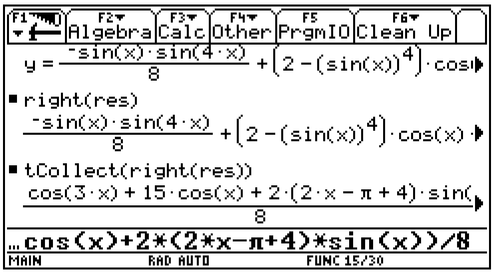
I believe that you are not surprised now that *MuPad* also delivers another form of the result – which proved to be wrong (satisfies the IVs, but does not satisfy the DE).

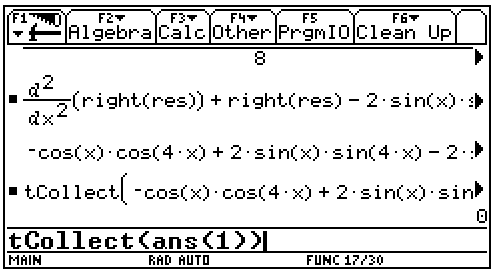
solve(ivp3)

$$\left\{\frac{23\cdot\cos(x)}{16} + \sin(x) + \frac{\cos(3\cdot x)}{32} + \frac{\cos(5\cdot x)}{32} - \frac{\pi\cdot\sin(x)}{4} + \frac{x\cdot\sin(x)}{2}\right\}$$

Finally I apply tCollect on the *DEQME*-output and receive a new appearance (right).

Then I substitute into the given equation and calculate the difference of the left side of the equation and the expected right side and hope that the result would be 0.





Both next problems read originally as follows:

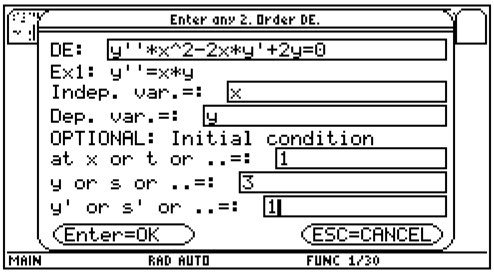
Verify that y_1 and y_2 are solutions of the DE. Then find a particular solution of the form $y = c_1\,y_1 + c_2\,y_2$ that satisfies the given initial conditions.

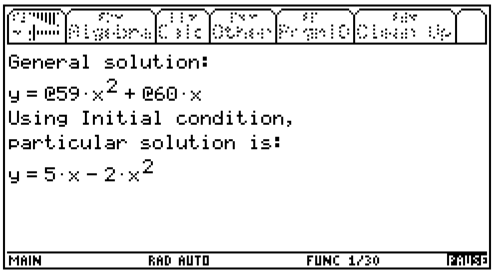
(4) $\quad x^2y'' - 2x\,y' + 2y = 0;\ \ y_1 = x,\ y_2 = x^2;\ y(1) = 3,\ y'(1) = 1$

(5) $\quad x^2y'' + 2x\,y' - 6y = 0;\ \ y_1 = x^2,\ y_2 = x^{-3};\ y(2) = 10,\ y'(2) = 15$

Example (4)

I try solving the DE and leave the check of the solutions y_1 and y_2 for later.





Again we don't encounter any problem using *DEQME*!

The next page shows the DE solved by *DERIVE*, *WIRIS*, *wxMaxima*, and *MuPad*:

$$\text{DSOLVE2\_IV}\left(-\frac{2}{x}, \frac{2}{x^2}, 0, x, 1, 3, 1\right) = 5\cdot x - 2\cdot x^2$$

WIRIS:

Example (4)

$\text{solve}(x^2\cdot y''(x)-2x\cdot y'(x)+2y(x)=0, y(1)=3, y'(1)=1) \rightarrow \{\{y(x)=-2\cdot e^{2\cdot \ln(x)}+5\cdot x\}\}$

$\text{simplify}(y(x)=-2\cdot e^{2\cdot \ln(x)}+5\cdot x) \rightarrow y(x)=-2\cdot x^2+5\cdot x$

WIRIS needs again an extra simplification for $e^{\ln x} = x$. Then see *wxMaxima* …

```
eqn_4:'x^2*'diff(y,x,2)-2*x*'diff(y,x,1)+2*y=0;
```

$$x^2\left(\frac{d^2}{d\,x^2}y\right) - 2\,x\left(\frac{d}{d\,x}y\right) + 2\,y = 0$$

```
ode2(%o11,y,x);
```

$$y = \%k1\,x^2 + \%k2\,x$$

```
ic2(%o12,x=1,y=3,'diff(y,x)=1);
```

$$y = 5\,x - 2\,x^2$$

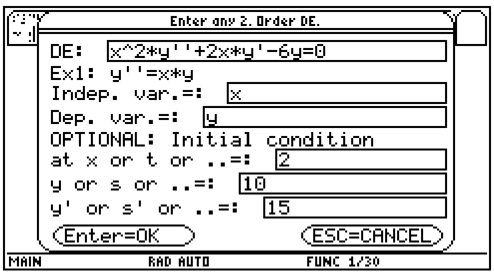
… followed by *MuPAD* (compact form of the procedure performed on page 26):

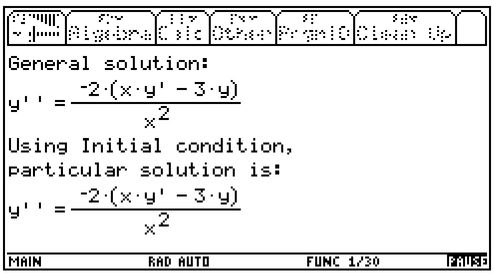
```
solve(ode({x^2*y''(x)-2*x*y'(x)+2*y(x)=0,y(1)=3,y'(1)=1},y(x)))
```

$$\{5\cdot x - 2\cdot x^2\}$$

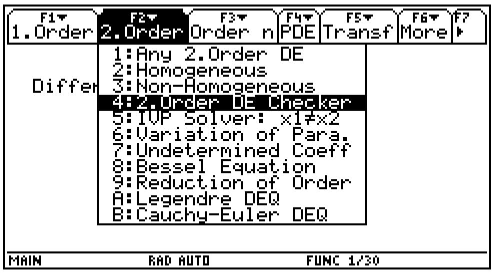
This runs pretty well, let´s try the next example which looks very similar.

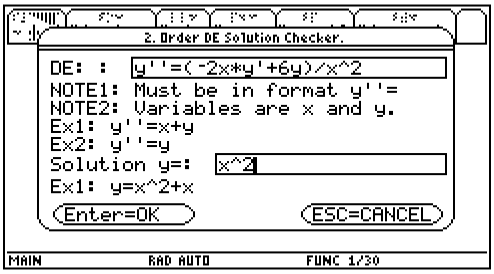
Example (5)

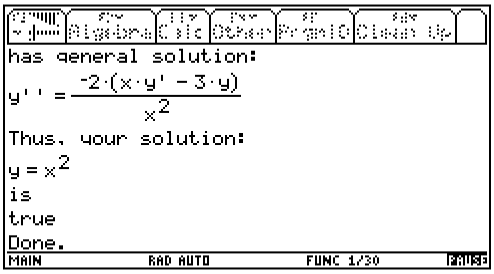


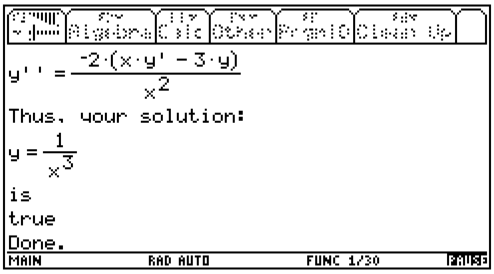


Interestingly this DE cannot be solved although it looks very similar to Example (4). Maybe that it doesn´t have any solution? I try the 2. Order DE Checker offered in Option 4 for y_1 and y_2:









y_1 and y_2 are solutions – as expected. So, we have solved the problem given in the textbook. I leave this DE for a later treatment and will look how DERIVE and other Computeralgebra Systems are performing. Doubtless I am starting again with DERIVE:

#2: $\text{DSOLVE2\_IV}\left(\dfrac{2}{x}, -\dfrac{2}{x^2}, 0, x, 2, 10, 15\right) = \text{inapplicable}$

I was not very much surprised about #2. The TI's algorithms are very close related with the *DERIVE* algorithms. (David Stoutemyer implemented the DE-package in *DERIVE* and the TIs as well.)

I checked the validity of y_1 and y_2 as solutions with *DERIVE*. Any linear combination of y_1 and y_2 should also form a solution, hence:

$\text{sol}(x) := c1\cdot x^2 + c2\cdot x^{-3}$

$x^2\cdot\text{sol}''(x) + 2\cdot x\cdot\text{sol}'(x) - 6\cdot\text{sol}(x) = 0$

So we are very curious what other systems will answer. Our next candidate is *WIRIS*:

Example (5)

$\text{prob5} := \text{solve}(x^2\cdot y''(x)+2x\cdot y'(x)-6\cdot y(x)=0, y(2)=10, y'(2)=15);$

$\text{prob5} \longrightarrow \{\{y(x)=-16.\cdot e^{-3.\cdot\ln(x)}+3.\cdot e^{2.\cdot\ln(x)}\}\}$

$\text{simplify}(y(x)=-16\cdot e^{-3\cdot\ln(x)}+3\cdot e^{2\cdot\ln(x)}) \longrightarrow y(x)=\dfrac{3\cdot x^5-16}{x^3}$

We obtain for $x > 0$ $y(x) = -16x^{-3} + 3x^2$. Then I try a compact form with *wxMaxima*.

```
ic2(ode2(x^2*'diff(y,x,2)+2*x*'diff(y,x,1)-6*y=0,y,x),x=2,y=10,'diff(y,x)=15);
```

$y = 3\,x^2 - \dfrac{16}{x^3}$

MuPad delivers the same solution without any problems. I don´t know why *DERIVE* refuses solving this equation, do you know?

Example (6)

Find the general solution of $x^2 y'' - 3x\,y' + 8y = x^2 + 2x$ for $x > 0$.

If $y(x)$ is a solution for $x > 0$ then $y(-x)$ is a solution for $x < 0$.

The “trick” is applying the substitution $x = e^s$. Then $u(s) = y(e^s) = y(x) = u(\log x)$.

The following is a useful application of the chain rule – for the students.

$$x = e^s \text{and } y(x) = y(e^s) = u(s)$$

$$u'(s) = y'(e^s)\cdot e^s = y'\cdot x$$

$$u''(s) = y''(e^s)\cdot e^s \cdot e^s + y'(e^s)\cdot e^s = y''\cdot e^{2s} + y'\cdot e^s = y''\cdot x^2 + y'\cdot x$$

hence

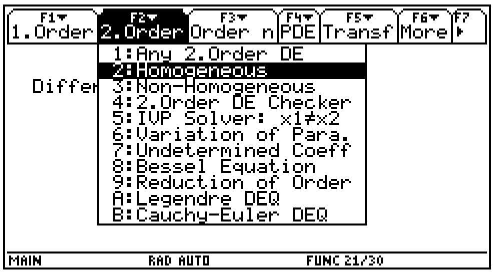
$$y''(x)\cdot x^2 = u''(s) - u'(s) \quad \text{and} \quad y'(x)\cdot x = u'(s)$$

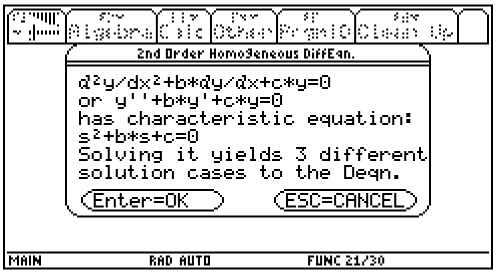
The given differential equation reads now (in variables *s* und *u*):

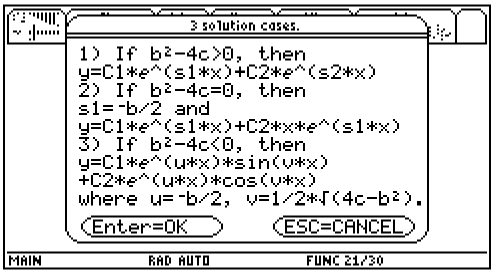
$$u'' - u' - 3u' + 8u = e^{2s} + 2e^s$$

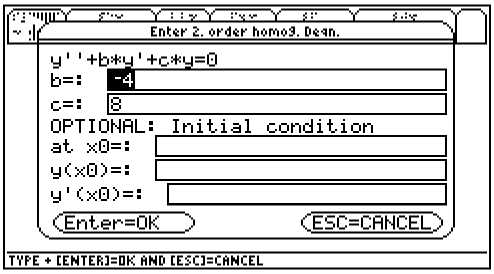
$$u'' - 4u' + 8u = e^{2s} + 2e^s$$

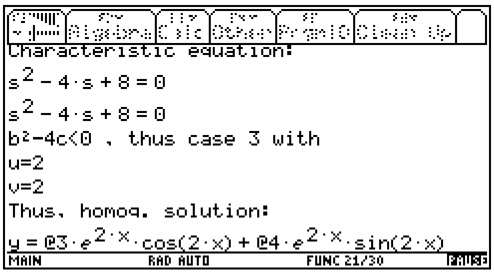
This is an nonhomogeneous ODE of 2nd order with constant coefficients. At first we have to find the general solution of the respective homogeneous DE. We ask *DEQME*:









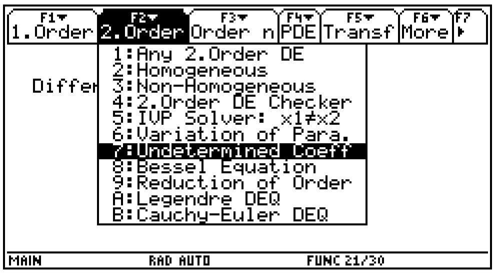


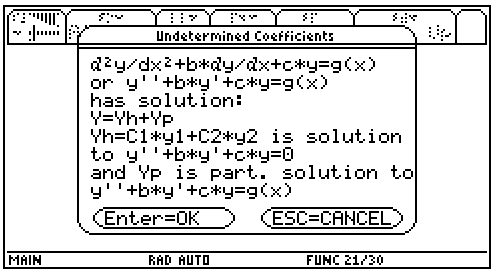
As we are restricted to *y* and *x* in this option, we have to rewrite the solution as

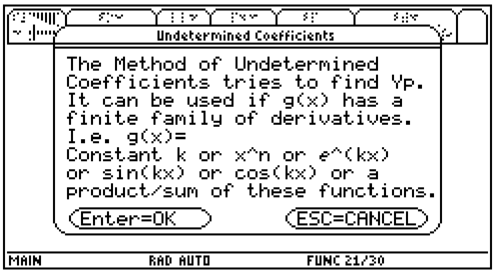
$$u(s) = c_1 \cdot e^{2s}\cos(2s) + c_2 \cdot e^{2s}\sin(2s).$$

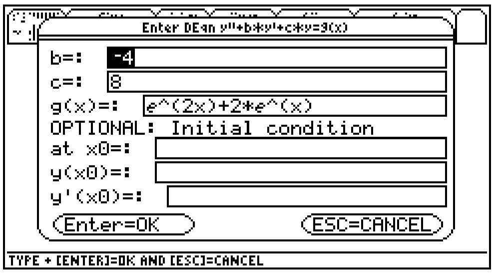
We need a particular solution to accomplish the solution for the nonhomogeneous equation.

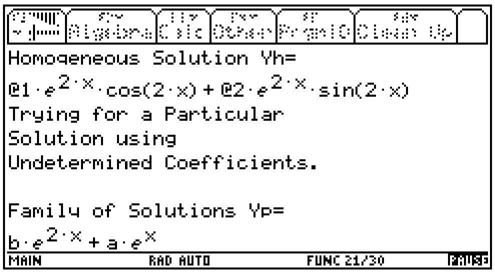
The method of undetermined coefficients seems to be appropriate. Again we ask *DEQME* for information and support.

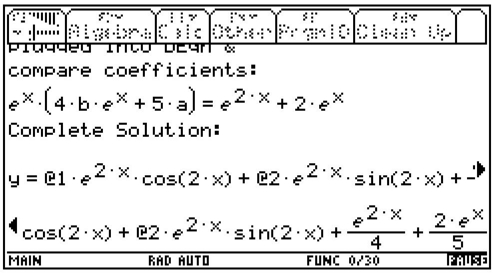












Now we have solved the (u,s)-DE which must be transformed back to the $y(x)$-function:

$x = e^s \leftrightarrow s = \log x$

(u,s): $\quad u(s) = c_1 \cdot e^{2s}\cos(2s) + c_2 \cdot e^{2s}\sin(2s) + \dfrac{e^{2s}}{4} + \dfrac{2e^s}{5}$

(y,x): $\quad y(x) = c_1 \cdot x^2\cos(2\log x) + c_2 \cdot x^2\sin(2\log x) + \dfrac{x^2}{4} + \dfrac{2x}{5}.$

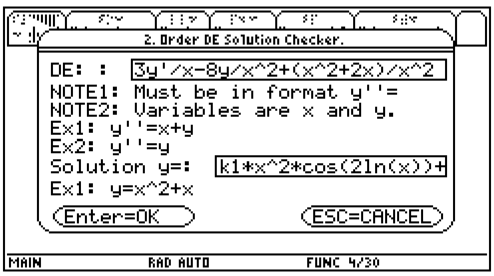
I regret having to admit that *DERIVE* is unable to solve this equation, so we will try with *wxMaxima*:

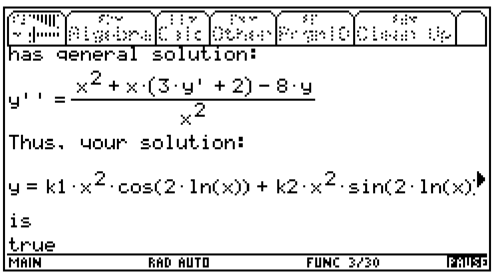
```
ode2(x^2*'diff(y,x,2)-3*x*'diff(y,x,1)+8*y=x^2+2*x,y,x);
```

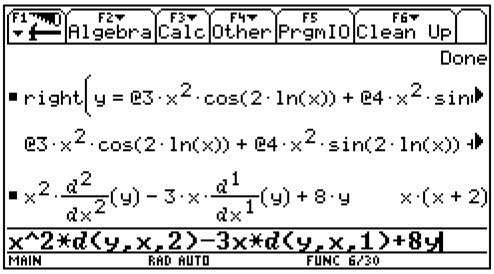
$$y = \frac{(5x^2+8x)\sin(2\log(x))^2 + (5x^2+8x)\cos(2\log(x))^2}{20} + x^2(\%k1\sin(2\log(x)) + \%k2\cos(2\log(x)))$$

I used the occasion to demonstrate Options 2 and 3 of the F2-menu.

We can apply the *DEQME* built-in DE Checker (Option 4) or do it on our own in the Home Screen (see below) for checking the solution.







As you might guess I was not really satisfied about *DERIVE*'s inability solving one or the other *Euler Equation.* So look at this:

(7) $x^2 y'' - 3x\,y' + 8y = x^2 + 2x$

(8) $x^2 y'' + 2x\,y' - 6y = 0;\ y(2) = 10,\ y'(2) = 15$

(9) $x^2 y'' + 2x\,y' - 6y = 30x;\ y(2) = 10,\ y(6) = 15$

(10) $x^2 y'' + 2x\,y' - 6y = 30x;\ y(2) = 10,\ y'(6) = 15$

(11) $x^2 y'' + 2x\,y' - 6y = 30x;\ y'(2) = 10,\ y'(6) = 15$

All of them cannot be solved using DSOLVE2, DSOLVE2_IV and DSOLVE2_BV respectively.
My EULER-functions do a better job.

Examples (7) – (11)

$$\text{DSOLVE2}\left(-\frac{3}{x}, \frac{8}{x^2}, \frac{x^2 + 2\cdot x}{x^2}\right) = \text{inapplicable}$$

but now

$$\text{EULER2}\left(-\frac{3}{x}, \frac{8}{x^2}, \frac{x^2 + 2\cdot x}{x^2}\right) = c1\cdot x^2\cdot \text{COS}(2\cdot \text{LN}(x)) + c2\cdot x^2\cdot \text{SIN}(2\cdot \text{LN}(x)) + \frac{x^2}{4} + \frac{2\cdot x}{5}$$

$$\text{EULER2\_IV}\left(\frac{2}{x}, -\frac{6}{x^2}, 0, x, 2, 10, 15\right) = 3\cdot x^2 - \frac{16}{x^3}$$

Parameter #9 in EULER2_BV (= k) = 0 (= default) for y0=y(x0) and y1 = y(x1)
= 1 for y0 = y(x0) and y1 = y'(x1)
= 2 for y0 = y'(x0) and y1 = y'(x1)

$$\text{EULER2\_BV}\left(\frac{2}{x}, -\frac{6}{x^2}, \frac{30}{x}, x, 2, 10, 6, 15\right) = \frac{145\cdot x^2}{88} - \frac{15\cdot x}{2} + \frac{1620}{11\cdot x^3}$$

$$\text{EULER2\_BV}\left(\frac{2}{x}, -\frac{6}{x^2}, \frac{30}{x}, x, 2, 10, 6, 15, 1\right) = \frac{310\cdot x^2}{163} - \frac{15\cdot x}{2} + \frac{22680}{163\cdot x^3}$$

$$\text{EULER2\_BV}\left(\frac{2}{x}, -\frac{6}{x^2}, \frac{30}{x}, x, 2, 10, 6, 15, 2\right) = \frac{1805\cdot x^2}{968} - \frac{15\cdot x}{2} - \frac{6480}{121\cdot x^3}$$

one check:

$$f(x) := \frac{1805\cdot x^2}{968} - \frac{15\cdot x}{2} - \frac{6480}{121\cdot x^3}$$

$$x^2\cdot f''(x) + 2\cdot x\cdot f'(x) - 6\cdot f(x) = 30\cdot x$$

We will ask *DEQME* to solve example (8) after having recognized that (8) is of *Euler* type on page 43 investigating Option B: Cauchy-Euler DEQ.

wxMaxima confirms the solution of boundary value problem (9):

```
bc2(ode2(x^2*'diff(y,x,2)+2*x*'diff(y,x)-6*y=30*x,y,x),x=2,y=10,x=6,y=15);
```

$$y = \frac{145\,x^2}{88} - \frac{15\,x}{2} + \frac{1620}{11\,x^3}$$

muPAD says that I am right with my solutions for (9), (10) and (11), thank you very much, indeed.

```
solve(ode({x^2*y''(x)+2*x*y'(x)-6*y(x)=30*x,y(2)=10,y(6)=15},y(x)))
```

$$\left\{\frac{145\cdot x^2}{88}-\frac{15\cdot x}{2}+\frac{1620}{11\cdot x^3}\right\}$$

```
solve(ode({x^2*y''(x)+2*x*y'(x)-6*y(x)=30*x,y(2)=10,y'(6)=15},y(x)))
```

$$\left\{\frac{310\cdot x^2}{163}-\frac{15\cdot x}{2}+\frac{22680}{163\cdot x^3}\right\}$$

```
solve(ode({x^2*y''(x)+2*x*y'(x)-6*y(x)=30*x,y'(2)=10,y'(6)=15},y(x)))
```

$$\left\{\frac{1805\cdot x^2}{968}-\frac{15\cdot x}{2}-\frac{6480}{121\cdot x^3}\right\}$$

These are two of my "home made" EULER-functions for DERIVE:

```
EULER2(p, q, r, x, c1, c2, ans) :=
  Prog
    ans := DSOLVE2(p·x - 1, q·x^2, SUBST(r·x^2, x, e^x), x, c1, c2)
    If ¬ STRING?(ans)
       LIM(ans, x, LN(x))
       "inapplicable"

EULER2_IV(p, q, r, x, x0, y0, v0, ans, v, s) :=
  Prog
    ans := DSOLVE2(p·x - 1, q·x^2, SUBST(r·x^2, x, e^x), x, c1, c2)
    If ¬ STRING?(ans)
       Prog
         v := ∂(ans, x)
         s := (SOLUTIONS([LIM(ans, x, x0) = y0, LIM(v, x, x0) = v0], [c1, c2]))↓1
         SUBST(ans, [c1, c2], s)
       "inapplicable"
```

I used these functions to solve problems (9) to (11).

I don´t copy the Boundary Value function EULER2_BV here to save space. Its syntax is as follows:

EULER2_BV(p, q, r, x, x1, y1, x2, y2, k)

with $k = 0$ (by default): given are $(x_1, y_1 = y(x_1))$ and $(x_2, y_2 = y(x_2))$
with $k = 1$: given are $(x_1, y_1 = y(x_1))$ and $(x_2, y_2 = y'(x_2))$ and
with $k = 2$: given are $(x_1, y_1 = y'(x_1))$ and $(x_2, y_2 = y'(x_2))$.

WIRIS cannot solve nonhomogeneous *Euler* DEs.

This is function *euler2* for the *TI-Nspire*:

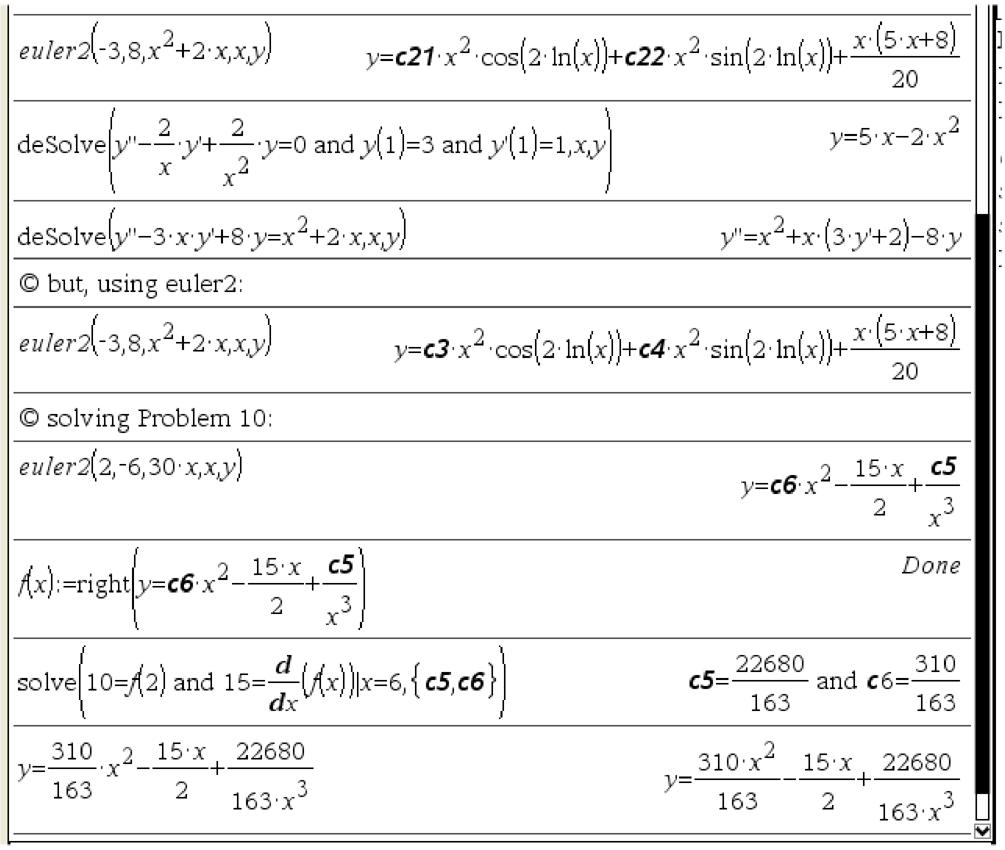
$\text{deSolve}\left(y''-\frac{2}{x}\cdot y'+\frac{2}{x^2}\cdot y=0,x,y\right)$	$y=c1\cdot x^2+c2\cdot x$
$\text{deSolve}\left(y''+\frac{2}{x}\cdot y'-\frac{6}{x^2}\cdot y=0,x,y\right)$	$y''=\frac{-2\cdot(x\cdot y'-3\cdot y)}{x^2}$
$euler2(-2,2,0,x,y)$	$y=c16\cdot x^2+c15\cdot x$
$euler2(2,-6,0,x,y)$	$y=c20\cdot x^2+\frac{c19}{x^3}$

euler2 3/4

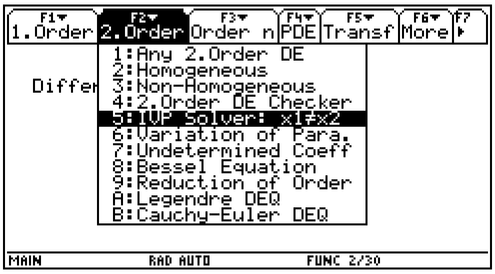
```
Define euler2(p,q,r,x,y)=
Func
Local k1,k2,sol_,eq
eq:=y''+(p-1)·y'+q·y=r|x=e^t
sol_:=deSolve(eq,t,y)
sol_|t=ln(x)
EndFunc
```

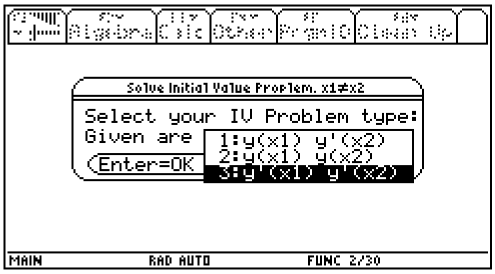
Nspire has the same problems solving *Euler* DEs as the TI-92 PLUS and the Voyage 200 as well.

The screen shot on the next page shows some examples worked with *TI-Nspire* applying *euler2*.



DEQME supports initial value and boundary value problems as well in Option 5:



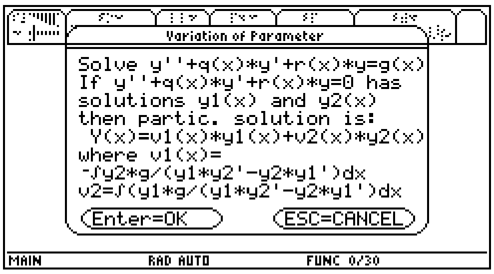


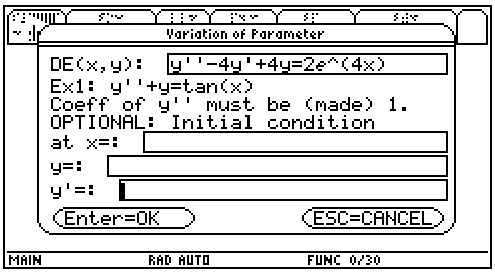
It works pretty well for all 2nd order ODEs which can be solved by the TI's DE-Solver.

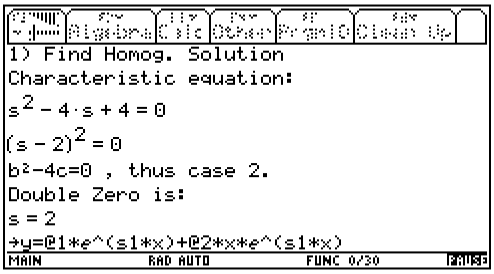
Proceeding investigating menu F2 I reach Option 6: Variation of Parameters. *DEQME* provides stepwise demonstrating this standard algorithm. I'll show two examples:

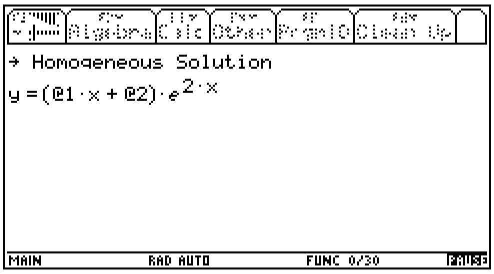
(12) $y'' - 4y' + 4y = 2e^{4x}$

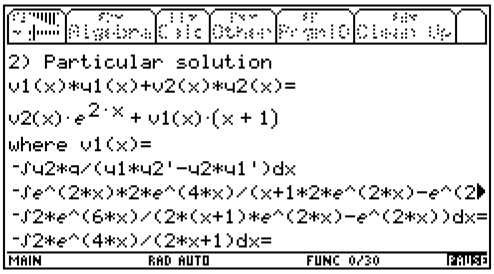
(13) $y'' - 9y = \sinh(2x);\ y(1) = 1,\ y'(1) = -0.5$

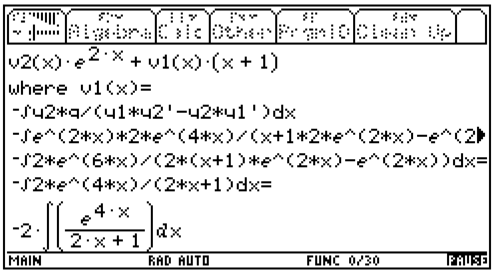


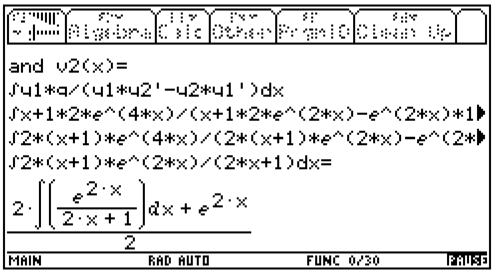


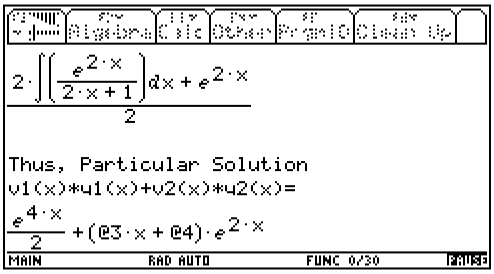












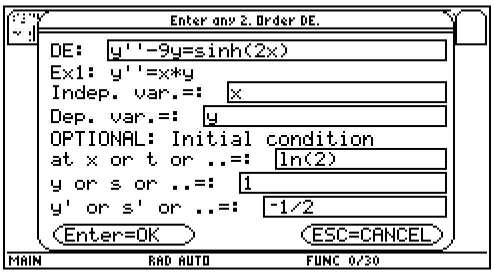
Maybe that students are happy with this, notify the result and proceed to the next example. I am not happy, because I can see that both integrals are not calculated. So this method fails! At the other hand there is a result, which says that the internal DE Solver finds the particular solution! What is my conclusion? What should the students conclude? There must be another method which should be more appropriate. Try in this case the Method of Undetermined Constants!

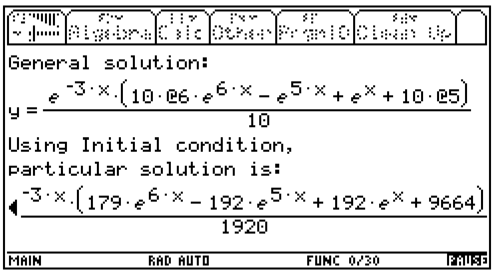
DERIVE returns - without stepwise simplification:

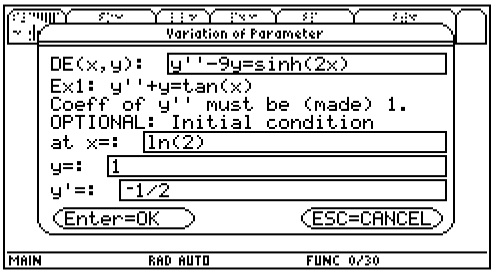
$$\text{DSOLVE2}(-4,\ 4,\ 2\cdot e^{4\cdot x}) = \frac{e^{4\cdot x}}{2} + e^{2\cdot x}\cdot(c2\cdot x + c1)$$

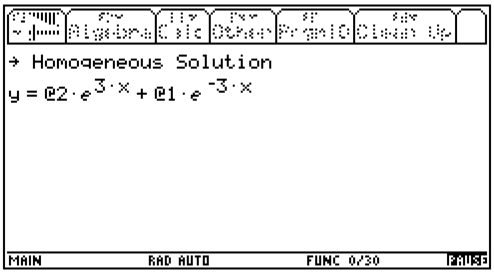
I recommend applying *DERIVE´*s Stepwise Simplification of this Solving procedure, it is an experience following the 30 “steps”. You will learn a lot about *DERIVE´*s “thinking”.

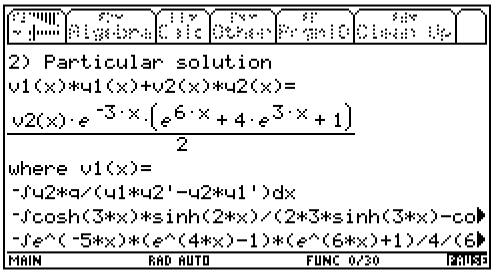
I am trying Parameter Variation once more solving example (13):

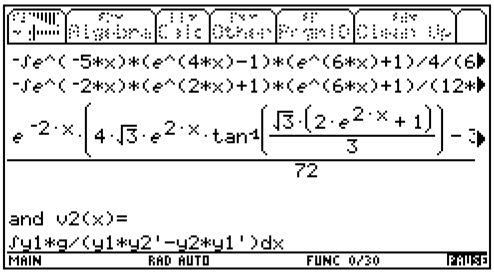


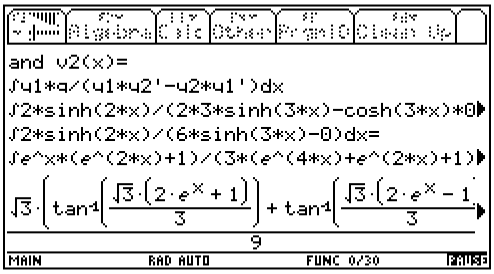


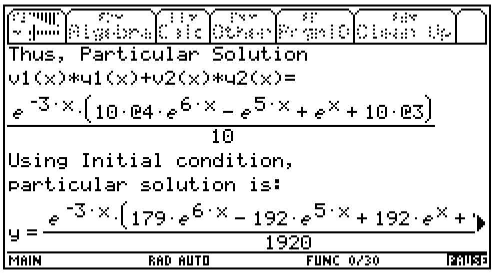












Compare with *DERIVE* followed by *wxMaxima*:

```
DSOLVE2_IV(0, -9, SINH(2·x), x, LN(2), 1, -0.5)
```

$$\frac{179\cdot e^{3\cdot x}}{1920}-\frac{e^{2\cdot x}}{10}+\frac{e^{-2\cdot x}}{10}+\frac{151\cdot e^{-3\cdot x}}{30}$$

```
ic2(ode2('diff(y,x,2)-9*y=sinh(2*x),y,x),x=log(2),y=1,'diff(y,x)=-1/2);
```

$$y=-\frac{\%e^{-2x}\left(\%e^{4x}-1\right)}{10}+\frac{179\,\%e^{3x}}{1920}+\frac{151\,\%e^{-3x}}{30}$$

F2 Option 7: Undetermined Coeff(icients) was treated on page 33.

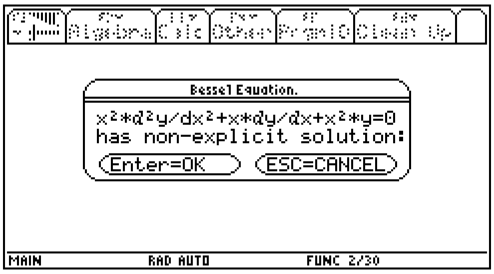
Options 6 and 7 are really nice and students and teachers as well might appreciate this tool. It could be a challenge to implement this "stepwise simplification" for *TI-Nspire* or *DERIVE* or other Computer Algebra Systems – as stand alone programs.

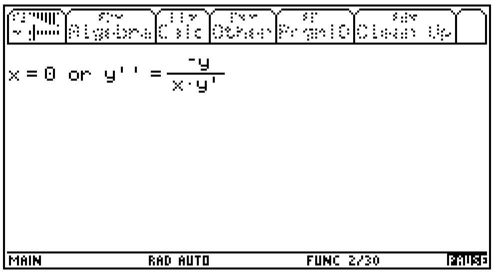
So we can proceed to Option 8: Bessel Equation.

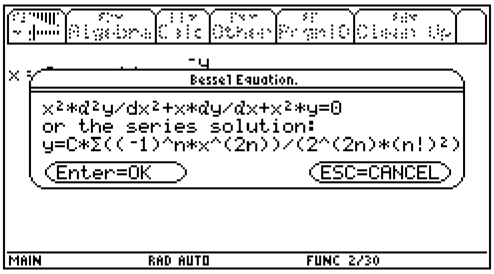
DEQME gives a short explanation of how the *Bessel Equation* looks like. The general form of the *Bessel* DE is:

$$x^2y''+xy'+(x^2-m^2)y=0.$$

I learned from the text books that the *Bessel* DE and *Legendre Equation* (Option A) appear when solving Partial Differential Equations (*Laplace Equation*).
Obviously *DEQME* shows only the – most important – case for $m = 0$.







See *MuPad*'s answer:

```
solve(ode(x^2*y''(x)+x*y'(x)+x^2*y(x)=0,y(x)))
Warning: Only Q-solvable exponential solutions will be found! [ode::secondOrder]
```

$\{C2 \cdot J_0(x) + C3 \cdot Y_0(x)\}$

and *wxMaxima*'s answer:

```
(%i30) ode2(x^2*'diff(y,x,2)+x*'diff(y,x)+x^2*y=0,y,x);
(%o30) y = bessel_y(0 , x)%k2 + bessel_j(0 , x)%k1

(%i31) ode2(x^2*'diff(y,x,2)+x*'diff(y,x)+(x^2-4)*y=0,y,x);
(%o31) y = bessel_y(2 , x)%k2 + bessel_j(2 , x)%k1

(%i32) ode2(x^2*'diff(y,x,2)+x*'diff(y,x)+(x^2-3)*y=0,y,x);
is √3 an integer? type y or n.
```

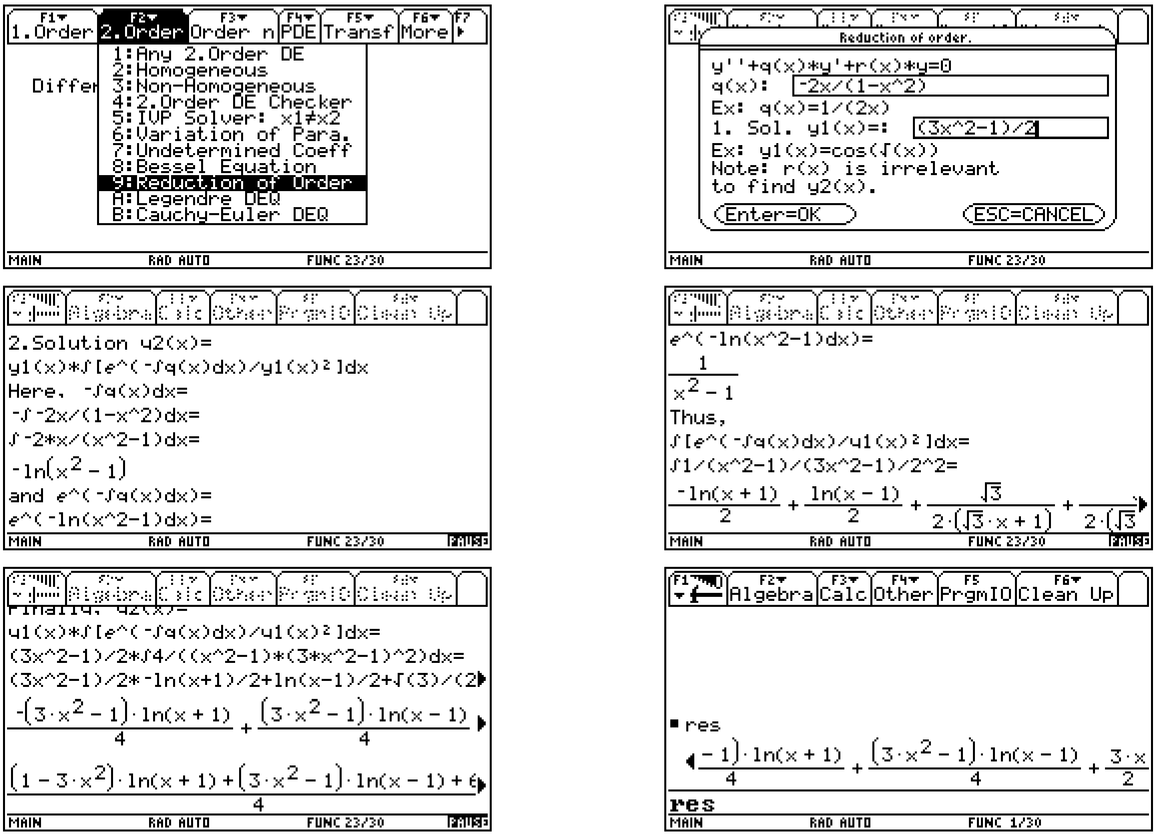
The solution contains the so called *Bessel functions* (see DNLs #18 and #34). *DERIVE* provides a utility file `BesselFunctions.mth`. *Bessel functions* can also be found in `SpecialFunctions.dfw` and `SpecialFunctions.mth` provided in the Users Folder.

After this short trip into the world of higher mathematics I will return to easier fields.

When I found F2 Option 9: Reduction of Order I – again – consulted my text books on ODEs and didn´t find anything matching with this option. I didn´t find one single problem similar to (14) or (15). Given is a 2nd order ODE and one of its solutions. Find the second one.

(14) $y''(1-x^2) - 2x\,y' + 6y = 0;\ \ y_1 = \dfrac{3x^2-1}{2}$

(15) $y'' - 2y' - 3y = 6;\ \ y_1 = e^{3x}$

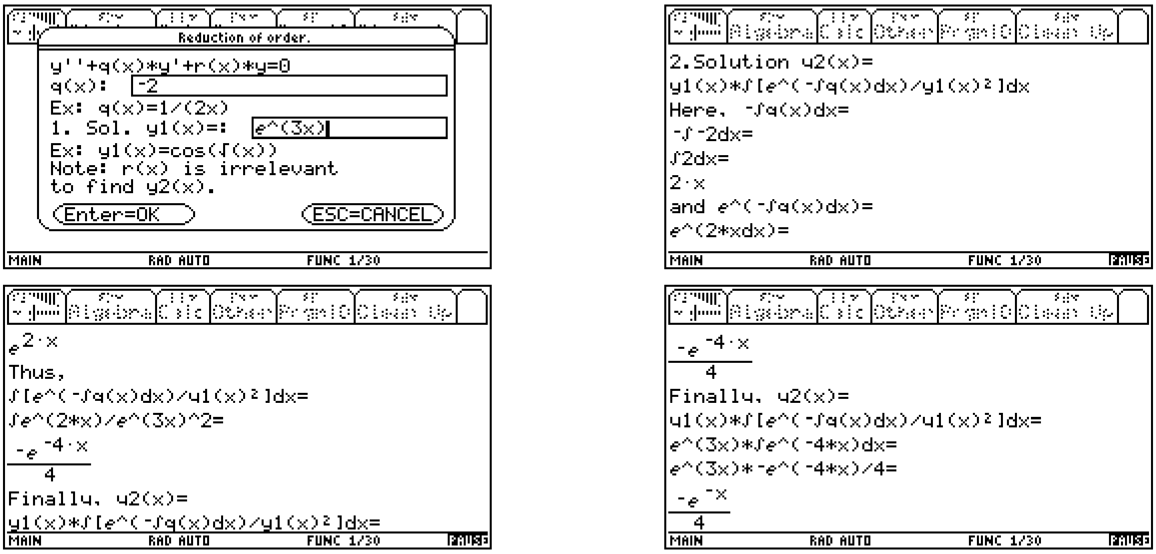


I check the validity of the complete solution – and I can be satisfied. *DEQME* does a good job.

$$\text{loes}(x) := c1\cdot\frac{3\cdot x^2 - 1}{2} + c2\cdot\left(\frac{(3\cdot x^2 - 1)\cdot \text{LN}\left(\frac{x-1}{x+1}\right)}{4} + \frac{3\cdot x}{2}\right)$$

$$\text{loes}''(x)\cdot(1 - x^2) - 2\cdot x\cdot \text{loes}'(x) + 6\cdot \text{loes}(x) = 0$$

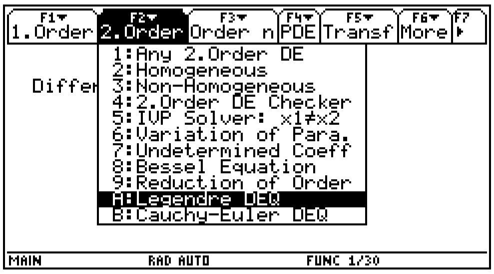
Example (15)

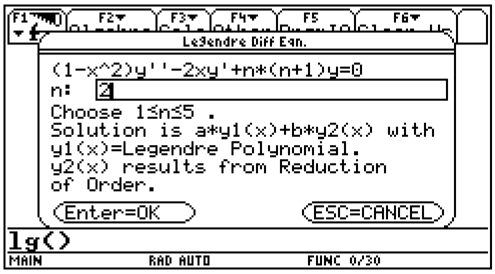


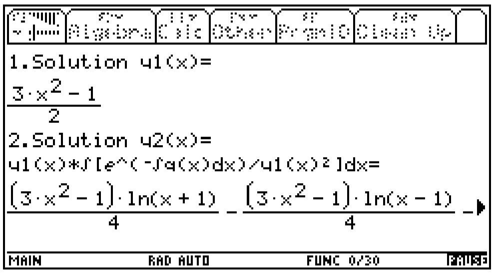
The complete solution of the homogeneous equation is $c_1 e^{3x} + c_2 e^{-x}$. Check it!

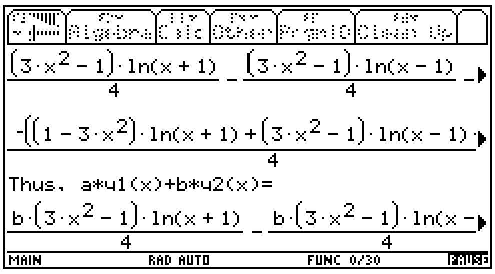
I mentioned the *Legendre Equation* earlier. Option A treats this kind of 2^{nd} order ODE.

(16) *Legendre* DE: $(1-x^2)y''-2xy'+n(n+1)=0;\ n\in N$.









Please compare with (14) from above. Now you might find an explanation for my choice of y_1.
The Legendre Polynomials for $n\in N$ can be produced by the "Formula of Rodrigues":

$$L^{(n)}=\frac{1}{2^n\,n!}\left(\frac{d}{dx}\right)^n(x^2-1)^n.$$

$$\text{VECTOR}\left(\frac{1}{2^k\cdot k!}\cdot\left(\frac{d}{dx}\right)^k(x^2-1)^k,\ k,\ 0,\ 5\right)$$

$$\left[1,\ x,\ \frac{3\cdot x^2-1}{2},\ \frac{x\cdot(5\cdot x^2-3)}{2},\ \frac{35\cdot x^4-30\cdot x^2+3}{8},\ \frac{x\cdot(63\cdot x^4-70\cdot x^2+15)}{8}\right]$$

As calculation times for finding the *Legendre polynomials* increase enormously on the TIs, it is restricted for $n\le 5$.

MuPAD:

```
solve(ode((1-x^2)*y''(x)-2*x*y'(x)+2*y(x)=0,y(x)))
```

$$\left\{C6\cdot x+C5\cdot\left(\frac{x\cdot\ln(x-1)}{2}-\frac{x\cdot\ln(x+1)}{2}+1\right)\right\}$$

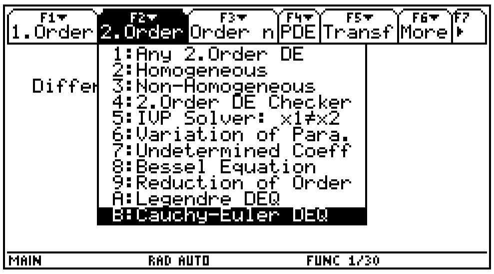
```
solve(ode((1-x^2)*y''(x)-2*x*y'(x)+6*y(x)=0,y(x)))
```

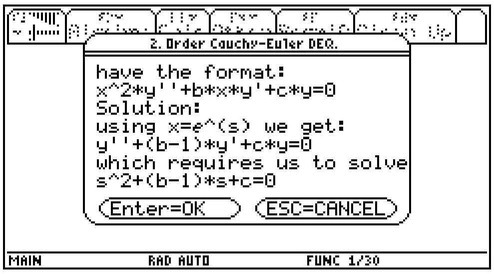
$$\left\{C3\cdot\left(x^2-\frac{1}{3}\right)+C2\cdot\left(\frac{9\cdot x}{4}-\frac{3\cdot\ln(x-1)}{8}+\frac{3\cdot\ln(x+1)}{8}+\frac{9\cdot x^2\cdot\ln(x-1)}{8}-\frac{9\cdot x^2\cdot\ln(x+1)}{8}\right)\right\}$$

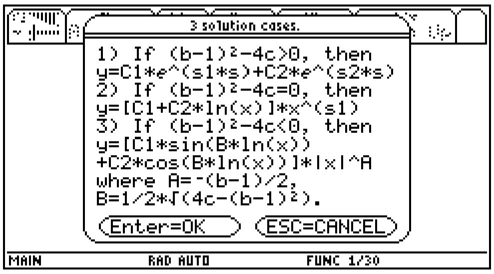
(Nice job for the students: Compare the result with the result from page 40, Problem (14)!)

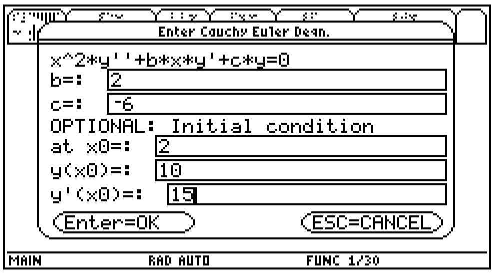
By the way, the *LDE* – algorithm works also for $n = 0$.

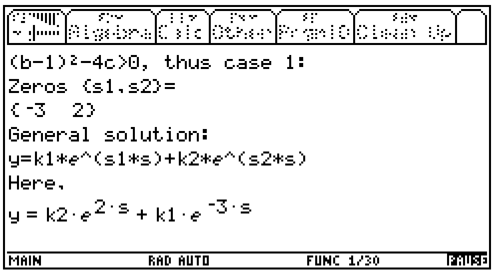
The last option doesn't need much space, because we talked about 2nd order *Euler Equations* and how to treat them on page 32. Option B does this in a very reduced form See example (8) treated with this option:

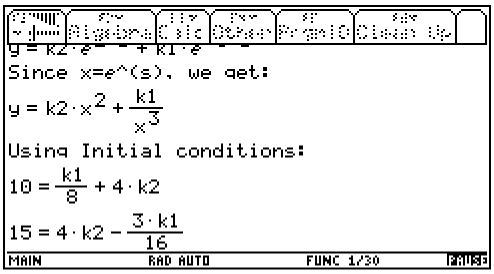


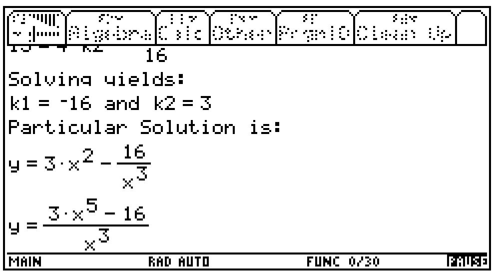












I am looking forward to investigating Menu F3 Order n containing the topics Linar+Const Coeff, 2×Linear System, X' = A ∗ X, X' = A ∗ X + F and Separable DE – and to "remember" a lot about Differential Equations again.

Differential Equations Made Easy for TI-89, 92+ and Voyage 200 is available from
http://www.ti89.com

You can find much information about *Legendre* and *Bessel DEs* at

http://mo.mathematik.uni-stuttgart.de/inhalt/beispiel/beispiel822/

http://en.wikiversity.org/wiki/Legendre_differential_equation

http://www.math.tugraz.at/~berglez/Math_C/Folien_10%28Potenzreihenans%29.pdf

References

[1] C.H.Edwards & D.E.Penney, *Differential Equations*, Prentice Hall 1996
[2] Günter/Kusimin, *Aufgabensammlung zur höheren Mathematik*, VEB Berlin 1964
[3] W.I.Smirnow, *Lehrgang der höheren Mathematik II*, VEB Berlin 1966
[4] E. Weisstein, *CRC Concise Encyclopedia of Mathematics*, CRC Press 1999
[5] *The Calculus Problem Solver*, REA 1985

Differential Equations "stepwise" with DERIVE and TI-Nspire

I liked the stepwise simplification in *DEQME* – users of *DEQME* appreciate this feature of *DEQME*, too. On page 39 I mentioned the "challenge" how to create a similar procedure with *DERIVE* or *TI-Nspire* or any other CAS-tool.

Here are my results for the homogenous 2[nd] order ODEs with constant coefficients:

The DERIVE Stepwise (without printing the progam):

"Stepwise solution for homogeneous 2nd order ODEs with constant coefficients.

y'' + b y' + c y = 0

hom2_IV(b,c,x0,y0,v0,x,y) with y(x0)=y0 and y'(x0)=v0.

Three examples:

y'' – 4y = 0

```
#1:    hom2_IV(0, -4)

Characteristic equation:

s = -2 ∨ s = 2

Discriminant = 16

Discriminant > 0 → two real solutions: s1 ≠ s2

[2, -2]

General solution: y = c1*e^(s1*x)+c2*e^(s2*x),

hence:
```

$$\#2:\quad y = c1\cdot e^{2\cdot x} + c2\cdot e^{-2\cdot x}$$

x''(t) – 6 x'(t) + 9 x(t) = 0

```
#3:    hom2_IV(-6, 9, t, x)

Characteristic equation:

s^2 - 6·s + 9 = 0

Discriminant = 0

Discriminant = 0 → one real double solution: s1 = s2

[3]

hence
```

$$\#4:\quad x = c1\cdot e^{3\cdot t} + c2\cdot t\cdot e^{3\cdot t}$$

$y'' - 4y' + 8y = 0,\ y(\pi/2) = 1,\ y'(\pi/2) = -1$

#5: $\text{hom2\_IV}\left(-4,\ 8,\ x,\ y,\ \frac{\pi}{2},\ 1,\ -1\right)$

Characteristic equation:

s^2 - 4·s + 8 = 0

Discriminant = -16

Discriminant < 0 → two complex solutions: s1 ≠ s2

[2 + 2·i, 2 - 2·i]

General solution: y = c1*e^(RE(s1)*x)*sin(IM(s1)*x)+c2*e^(RE(s2)*x)*cos(IM(s2)*x),

hence:

General solution:

y = e^(2·x)·(c2·COS(2·x) + c1·SIN(2·x))

Solving the Initial Value Problem:

1. equation: y(x0)=y0

c2 = - e^(-π)

2. equation: y(x0)=v0

2·c1 + 2·c2 = e^(-π)

Solutions for c1 and c2 are:

[[3·e^(-π)/2], [- e^(-π)]]

Solution

#6: $\left[\begin{array}{ll} \text{General solution} & y = e^{2\cdot x}\cdot(c2\cdot\text{COS}(2\cdot x) + c1\cdot\text{SIN}(2\cdot x)) \\ \text{Therefore, the solution is:} & y = e^{2\cdot x - \pi}\cdot\left(\frac{3\cdot\text{SIN}(2\cdot x)}{2} - \text{COS}(2\cdot x)\right) \end{array}\right]$

#7: $\text{DSOLVE2\_IV}\left(-4,\ 8,\ 0,\ x,\ \frac{\pi}{2},\ 1,\ -1\right) = e^{2\cdot x - \pi}\cdot\left(\frac{3\cdot\text{SIN}(2\cdot x)}{2} - \text{COS}(2\cdot x)\right)$

The “steps” printed in blue are created by using the DISPLAY-function. We don’t have a PAUSE-function like in the TI-89, 92 and Voyage 200 programming language. So I can show the steps in a form of report only.

Unfortunately I cannot enter “*DERIVE*’s interior” in order to make use of *DERIVE*´s stepwise calculation. As I mentioned ealier it is very interesting to apply the *DERIVE* steps on the *DERIVE* functions. Sometimes you might fail. I entered DSOLVE2_IV(-4, 8, 0,x,π/2,1,-1) and was stopped by “Memory exhausted” after three steps ….

Using the features of the latest version of *TI-Nspire* (there are rumors that release of version 3 will be soon) results also in a nice dialogue-driven commented output of the algorithm.

The Request-, Text-, and Disp-command are of importance. The values for b and c, and for x0, y(x0) and y'(x0) are entered in a dialogue box.

hom2_iv()

DE of form y'' + b*y' + c*y = 0

Enter b,c: 0,−4

hom. DE:

$y''-4\cdot y=0$

Enter Initialvalues x0, y(x0), v0=y'(x0)

ENTER 0 if none

Initialvalues x0,y(x0),y'(x0): 0

Characteristic Equation:

$s^2-4=0$

Discriminant =

16

Discriminant > 0 → two real solutions: s1 ≠ s2

$\{-2,2\}$

General solution is

$y=c2\cdot e^{2\cdot x}+c1\cdot e^{-2\cdot x}$

Done

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Done

hom2_iv()

DE of form y'' + b*y' + c*y = 0

Enter b,c: −4,8

hom. DE:

$y''-4\cdot y'+8\cdot y=0$

Enter Initialvalues x0, y(x0), v0=y'(x0)

ENTER 0 if none

Initialvalues x0,y(x0),y'(x0): pi/2,1,−1

Characteristic Equation:

$s^2-4\cdot s+8=0$

Discriminant =

-16

Discriminant < 0 → two complex solutions: s1 ≠ s2

$\{2+2\cdot i,2-2\cdot i\}$

General solution is

$y=c2\cdot e^{2\cdot x}\cdot \cos(2\cdot x)+c1\cdot e^{2\cdot x}\cdot \sin(2\cdot x)$

Solving the Initial Value Problem:

1. Equation: y(x0) = y0

$1=-c2\cdot e^{\pi}$

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2. Equation: y'(x0)=v0

$-1=-2\cdot c1\cdot e^{\pi}-2\cdot c2\cdot e^{\pi}$

Solutions for c1 and c2 are:

$c1=\frac{3}{2\cdot e^{\pi}}$ and $c2=\frac{-1}{e^{\pi}}$

Special solution is

$y=\frac{3\cdot (e^{x})^2\cdot \sin(2\cdot x)}{2\cdot e^{\pi}}-\frac{(e^{x})^2\cdot \cos(2\cdot x)}{e^{\pi}}$

Done

The screen shots show the Calculator Application.

This is the first part of the *TI-Nspire* program.

Below is the start of the *DERIVE* function.

Both programs can be downloaded from the DUG-website (contained in mth80.zip).

```
hom2_iv
Define hom2_iv()=
Prgm
Local co,ivs,d,sols,gen_sol,y_1,eq1,eq2,cs,x0,y0,v0
Disp "DE of form y'' + b*y' + c*y = 0"
RequestStr "Enter b,c:",co
co:="{"&co&"}":co:=expr(co)
b:=co[1]: c:=co[2]
Disp "hom. DE: "
Disp y''+b·y'+c·y=0
Disp "Enter Initialvalues x0, y(x0), v0=y'(x0)"
Disp "ENTER 0 if none"
RequestStr "Initialvalues x0,y(x0),y'(x0):",ivs
ivs:="{"&ivs&"}"
ivs:=expr(ivs)
d:=b^2-4·c
sols:=cZeros(s^2+b·s+c,s)
Disp "Characteristic Equation: "
Disp s^2+b·s+c=0
```

```
hom2_IV(b, c, x, y, x0 := i, y0, v0, d, s_, sols, gen_sol, sp_sol, c1, c2,
  Prog
    d := b^2 - 4·c
    sols := SOLUTIONS(s^2 + b·s + c = 0, s)
    DISPLAY("Characteristic equation:")
    s_ := '(s^2 + b·s + c = 0)
    DISPLAY(s_)
    "DISPLAY(s^2 + b·s + c = 0)"
    DISPLAY("Discriminant" = d)
    If d > 0
       Prog
         DISPLAY("Discriminant > 0 → two real solutions: s1 ≠ s2")
         DISPLAY(sols)
         DISPLAY("General solution: y = c1*e^(s1*x)+c2*e^(s2*x),")
         DISPLAY("hence:")
         gen_sol := y = c1·e^(x·sols↓1) + c2·e^(x·sols↓2)
    If d = 0
       Prog
         DISPLAY("Discriminant = 0 → one real double solution: s1 = s2")
         DISPLAY(sols)
         DISPLAY("General solution: y = c1*e^(s1*x)+c2*x*e^(s2*x),")
         DISPLAY("hence:")
         gen_sol := y = c1·e^(x·sols↓1) + c2·x·e^(x·sols↓1)
```